



Preserving Statistic Kendall's Orders

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Abstract

We deal with some properties of Kendall's statistic Orders. In particular, given two statistic variables we will prove when the order is preserving, not changing anything in the reference aleatory system. It is a rare situation so that we can consider the opposite situation in which some conditions are changing and the orders might change and not to be always preserved. Then, changing the definition of orders in the statistic initial variables, the implication could be still true but considering the changing of definition of statistic orders. Instead, even if in this conditions, we analysis whenever statistic orders is preserving as well. When initial conditions are more ridge in respect to final results, we will prove why they change the system and the orders are not preserving.

Keywords: Kendall's orders, Stochastic systems, Statistic orders

Introduction

A system, if it is not subjected to alteration of properties and initial configurations tends to preserve its characteristics and also on data, we have not significantly changing into initial configurations, then if we consider an initial order in the Kendall sense, it is preserving after a certain period of time. Even if we consider Kendall's orders as association rules among stochastic variables, they seems not to change and preserve the original configurations. In this paper, we want to describe when the Kendall's order is prevented and when it is not prevented. Kendall's order is used to study the distribution-free Mann-Kendall test that it is widely used for the assessment of significance of trends in many hydrologic and climatic time series, see [1] for further details. Then, the relationship between Spearman's ρ_n and Kendall's τ_n for the two extreme

order statistics $X(1)$ and $X(n)$ of n independent and identically distributed continuous random variables is expresses in [2]. They are also compared as linear orders in [3]. Kendall's plots are studying in [4], which they adapt the concept of probability plot to the detection of dependence of random variables. Finally, the Kendall's rank correlation coefficient is used for a stochastic ordering based on a decomposition of Kendall's Tau, in [5].

Main Results and Some Examples

In this section, we analyse all possible configurations in preventing Kendall's orders and some applying examples.

Theorem 1. If we consider two arrays

$X_1 = (x_{11}, \dots, x_{1n}) \leq X_2 = (x_{21}, \dots, x_{2n})$ and $Y_1 = (y_{11}, \dots, y_{1n}) \leq Y_2 = (y_{21}, \dots, y_{2n})$ then

$X_1 X_2 = (x_{11}x_{21}, \dots, x_{1n}x_{2n}) \leq Y_1 Y_2 = (y_{11}y_{21}, \dots, y_{1n}y_{2n})$, so that if we do not change the order in the final state, the system will preserve its conditions and the Kendall's order is preserving.

Theorem 2. If we consider two arrays $X_1 = (x_{11}, \dots, x_{1n}) \leq X_2 = (x_{21}, \dots, x_{2n})$ and

$Y_1 = (y_{11}, \dots, y_{1n}) \leq Y_2 = (y_{21}, \dots, y_{2n})$ then, $X_1 Y_1 = (x_{11}y_{11}, \dots, x_{1n}y_{1n}) \leq X_2 Y_2 = (x_{21}y_{21}, \dots, x_{2n}y_{2n})$

So that if we do not change the order in the final state, the system will preserve its conditions and the Kendall's order is preserving, also considering the associated rules.

Theorem 3. If we consider two arrays $X_1 = (x_{11}, \dots, x_{1n}) \leq X_2 = (x_{21}, \dots, x_{2n})$ and $Y_1 = (y_{11}, \dots, y_{1n}) \leq Y_2 = (y_{21}, \dots, y_{2n})$ then, $X_1 X_2 = (x_{11}x_{21}, \dots, x_{1n}x_{2n}) \geq Y_1 Y_2 = (y_{11}y_{21}, \dots, y_{1n}y_{2n})$

, so that if the significant change in the ordered initial data perturbate the final state, the system will not preserve its conditions and the Kendall's order is not preserving.

Theorem 4. If we consider two arrays $X_1 = (x_{11}, \dots, x_{1n}) \leq X_2 = (x_{21}, \dots, x_{2n})$ and $Y_1 = (y_{11}, \dots, y_{1n}) \leq Y_2 = (y_{21}, \dots, y_{2n})$ then, $X_1 X_2 = (x_{11}x_{21}, \dots, x_{1n}x_{2n}) \geq Y_1 Y_2 = (y_{11}y_{21}, \dots, y_{21}y_{2n})$

so that if we change the order in the final state, the system will not preserve its conditions and the Kendall's order is not preserving.

These results come from perturbations onto initial data and if they are consistent with fluctuations of initial conditions in the initial system, they have to change a lot and to change consistently all the final conditions. Instead, if the system does not change aleatory, it will preserve all defined statistic orders. In the first configuration, if external conditions do not alter the internal ones, also the final states will preserve statistic orders as defined initially.

The second configuration required a revisitation of initial definitions of statistic orders because small changes onto initial configurations perturbate the system and the final state which it requires a new definition that it is a small change onto the definition of the final set of ordered final system.

The third and fourth configurations are concerning a lot of change onto the physical system. Then, in the third configuration a big perturbation of initial data could alter the final configuration and in the fourth configuration as well, it is perturbing a lot and final status requires a change of final ordered system, as the final system is changing a lot.

Now, we focus on providing some examples of such configurations in game theory or general physical stochastic situations. Indeed, the first configuration corresponds to regular games in which players are not changing strategy of the game and no ones are losing its turn. The second configuration is the case in which some players had lost the play and it could expect one cycle of plays, then the order is newly established. When player's order is changing in the third and fourth configurations is because the game impose to invert the game order. Other reason is to calculate how many times this is happening during the games, but we leave this for future works.

In other physical stochastic situations, we can consider typical reaction networks, in which if external conditions do not change and no external forces can perturbate the system, we can reconvert to the first and second configuration of the initial system.

The third and fourth conditions of disordered systems are the results of interventions of external forces and the system is then altered a lot. In these situations strong reaction networks are studying among atoms because of external agents contribute to perturbate the initial physical system.

This study is adopting very well also in the study of transaction and processes of elaboration of data, because of all instantiations are subjected aleatory to the functionalities of systems of elaborations of data and can cause big interrupts during these processes of elaborations of data. These interruption could be classify based on the fourth configurations we described above.

Conclusion

We reported some few examples on how the Kendall's statistic order can be employed to study the game theory and physical movements that can perturbate and alter initial conditions, also onto given data set. Indeed, it is a reconditioned result that small perturbation onto initial data can not cause alterations onto final data whilst a big perturbation of initial states can cause a very big lost of information onto final data. These analysis are valid also for big processes of elaborations of various data set. We applied general facts of theory of data to Kendall's statistic orders.

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None

Conflict of Interest

No conflict of interest.

References

1. K. H. Hamed (2008) Exact distribution of the Mann–Kendall trend test statistic for persistent data. *Journal of Hydrology* 1(2): 86-94.
2. Y-P. Chen (2005) A Note on the Relationship between Spearman's ρ and Kendall's τ for Extreme Order Statistics. *Journal of Statistical Planning and Inference* 137(7): 2165-2171.
3. B. Monjardet (1998) On the comparison of the Spearman and Kendall metrics between linear orders. *Discrete Mathematics* 192(1-3): 281-292.
4. J-C. Boies, C. Genest (2003) Detecting Dependence With Kendall Plots. *The American Statistician*, pp.275-284.
5. P. Caperaa, A-L. Fougères, C. Genest (1997) A Stochastic Ordering Based on a Decomposition of Kendall's Tau. *Distributions with given Marginals and Moment Problems*.