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Research Article

Estimation Of Strength for Inhomogeneous Sample of the Porous Plate with Taking into Account of the Internal Heat Sources Power

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Abstract

In this paper we introduce on the first stage the results of investigation of wave propagation into inhomogeneous sample of porous ceramic plate with slowly varying refractive index. This approach considering the results, taking into account the resolving of wave equation for the plate polarized electromagnetic wave, which is transferring through the porous plate perpendicularity to the external surface. We are using the corresponding boundary conditions for the model parameters. For electromagnetic wave we are reviewing the transverse electrical (T.E.) as well, as the transverse magnetic (T.M.) mode, which are shortly named as the (T.E.M.) mode. It was be received the variants of solution for the approach of Wentzell-Kramers-Brilluon (W.K.B.) for the symmetric falling irradiation of the fixed power, which is generated on the surfaces of plate and the Jeffries [9] bounds was introduced, which takes the possibility to analyze the coefficients of transmission and reflection on the surfaces of plate. On the seconds stage the criterious relations for estimation of stretch for porous material with taking into account of the internal sources power which follows to crashing of the porous body are formulated.

Keywords: Porous media; Electromagnetic waves; Dielectric loss factor; The energy characteristics of interface layers; The boundaries of plasticity and strength; Criterion ratios

The scientific results

At investigation of propagation of electromagnetic waves of microwave frequency into porous wetted media the method of local space averaging was used [1], according to it for inhomogeneous (nonmagnetic) multiphase material with dielectric losses and low conductor properties [2] under frequency dispersion of dielectric permeability of components we are using the information about the effective generalized complex dynamic dielectric permeability (E.G.C.D.D.P.) [3]. Such dielectric permeability into approach of effective macroscopic field takes the possibility to determine the lineal response of not interacting between vise verse mezoscopic averaging volumes (R.E.V. - Representative Elementary Volume [1]) on the base of cluster approach, if into the considering object the harmonic disturbance is occurs as a reaction on the external plate monochromatic harmonically electromagnetic wave of microwave frequency. For modeling of the processes into object we introduce the dielectric permeability $\bar{\varepsilon}_{\omega}^{ef}\left(\vec{x},t\right)$ [3], which take a possibility to review the porous humidified media, as a homogeneous isotropic dielectric with continuously changed into coordinate space and time electro-physical properties (this is the continuous media or the field approach). Taking into account, that mutual displacement of phases joined with hydrodynamic (mechanical) moving of liquid or gas into porous skeleton is unsignifical on the distances of epy electromagnetic wave length, which interacts with body. Also, according to the slow varying conditions of volume characteristics of porous material we are considering the not important (light) time dependence of E.G.C.D.D.P. during the period of oscillation. Relaying to this the effective complex dielectric permeability $\overline{\varepsilon}_{\omega}^{ef}\left(\vec{x},t\right)$ is reviewing as slow varying function of coordinate and time. The equation of field from which we get the expression for the power of microwave heat sources is resolved abstractedly from the heat and mass transfer equations into modelled porous (humidified) material.

We are considering the porous media as composite concrete ceramics which consist the large number of interconnected pores with the little size and porous skeleton (S is the solid phase) [5]. We assume that the media of the object (body) is unsaturated, in particular, into pores exist the substance in the liquid (L) and gas like phase (G). The last one is the two-component mixture of atmospheric dry air and water vapor [3]. The pores into the fist approach are modelled by the spherical inclusions with linear radius R: the micro pores (R < 2 nm), mezzo pores (2nm <R < 50 nm) and macro pores (R > 50 nm) [6].

We are considering the solution of received into harmonically approach the equations [4] for electromagnetic field (E.M.F.), in which the such parameters are fulgurated: the Φ_{*}^{t} is the complex amplitude of the E.M.F. $\left(\vec{E},\vec{H}\right)$ into porous cell of considering media (here $\Phi=\mathrm{Re}\left[\Phi_{*}^{t}\right]$ is the real quantity of the complex amplitude) as well, as $\overline{\varepsilon}_{\omega}^{ef}\left(x_i,t\right)$ is the effective generalized complex dynamical dielectric permeability (E.G.C.D.D.P.) [3] (here the time t is into content of a parameter, by means of which the moving of phases according to deformation process of solid phase (skeleton) and flowing phases (liquid or gas) under influence of evaporation or condensation during heat and mass transfer are considered. The mentioned parameters we are considering the slowly (light) varying functions of coordinate. Then equations for the plate (transverse) polarized T.E.M. [7] monochromatic wave in the stretch of electric field $\vec{\mathrm{E}}_{*}^{t}$ into porous composite media [2] takes the form

$$\Delta\vec{\mathrm{E}}_{*}^{t}\left(\vec{x},t\right)+k_0^2\left[x,t\right]^2\vec{\mathrm{E}}_{*}^{t}\left(\vec{x},t\right)=0 \qquad (1)$$

Here $\overline{\varepsilon}_{\omega}^{ef}\left(\vec{x},t\right)$ is the effective generalized complex dynamical dielectric permittivity, $\overline{n}_{\ddot{u}}^{'}\left(\vec{x},\omega\right)=\overline{k}_{\ddot{u}}^{'}\left(\vec{x},t\right)/k_0=\sqrt{\overline{\varepsilon}_{\omega}^{ef}\left(\vec{x},t\right)}$ is the complex refractive index; $\overline{k}_{\ddot{u}}^{'}\left(\vec{x},t\right)$ is the wave vector into the porous (inhomogeneous) media; $k_0=\omega\sqrt{\mu_0\varepsilon_0}=\omega/c_0$ (here $c_0=1/\sqrt{\mu_0\varepsilon_0}$ is the velocity of light) is the wave vector of electromagnetic wave into vacuum, $\omega=2\pi\nu$ is the corner frequency of E.M.F. (ν is the linear frequency), μ_0 and ε_0 are correspondingly the magnetic and electric constants into vacuum.

The bounds between the measured $\overline{n}_{\ddot{u}}^{'}\left(x_i,\omega\right)$ and modeled $\overline{n}_{\omega}^{eff}\left(x,t\right)$ indexes we describe into approach of the effective field (E.M.A.) according to well known Bruggeman [3] formula

$$\sum_{\sigma}\theta_{\sigma}\frac{\varepsilon_c^{\sigma}\left(\omega\right)-\overline{\varepsilon}_{\omega}^{eff}}{\varepsilon_c^{\sigma}\left(\omega\right)+2\overline{\varepsilon}_{\omega}^{eff}}=0 \qquad (2)$$

here $\varepsilon_c^{\sigma}\left(\omega\right)=\varepsilon_c^{\prime\sigma}\left(\omega\right)-i\varepsilon_c^{\prime\prime\sigma}\left(\omega\right)$ is the static complex dielectric permittivity for $\sigma=\{S,L,G\}$ phase under E.M.F. frequency ω for investigated porous composite media, where is takes formally $\overline{n}_{\ddot{u}}^{'}\left(x_i,\omega\right)\equiv\overline{n}_{\omega}^{eff}\left(x,t\right)$ according to ergodicity hypotheses.

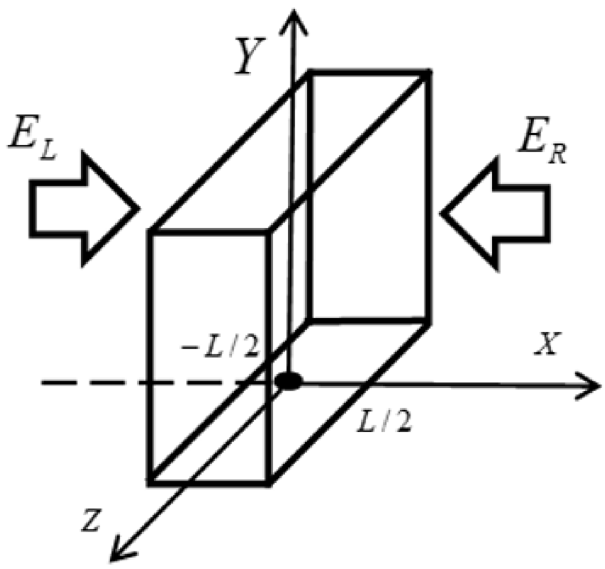

Figure 1: Symmetrical (at conditions of equality for amplitudes of electrical field) Irradiation of porous plate.

As a porous sample during numerical experiment, we review the porous wetted (unlimited) porous plate. By the selection of the direction of distribution for electromagnetic wave along the axis 0*X* at the fixed polarization along 0*Y* (axis of ordinate *Y*) (Figure1) the wave equation (2) takes [2] the following view

$$\frac{\partial^2 E_*^t(x)}{\partial x^2}+k_0^2\left[\overline{n}_\omega^{eff}(x,t)\right]^2 E_*^t(x)=0 \tag{3}$$

where E_*^t is the complex amplitude of electric field stretch along the axis 0*Y*.

We use the Wentzel-Kramers-Brillouin (W.K.B) approximation [9] and will look for a partial solution of equation (3) in the form $E_*^t(x)=A\exp\left[i\phi(x,t)\right]$, where *A* is the unknown complex constant and $\phi(x,t)$ is the the phase of an electromagnetic wave varies depending on coordinates and time. After substituting the specified expression for $E_*^t(x)$ into relation (3), we obtain the equation

$$\left(\frac{\partial\phi}{\partial x}\right)^2=k_0^2\left[\overline{n}_\omega^{eff}(x,t)\right]^2\left(1+i\frac{1}{k_0^2\left[\overline{n}_\omega^{eff}(x,t)\right]^2}\frac{\partial^2\phi}{\partial x^2}\right) \tag{4}$$

Let us assume that the last term in (4) can be neglected due to the weak phase variability perpendicular to the wave propagation front. Then the condition of weak variation in the coordinate space (for the variable x it is the one-dimensional case) according to the relation (4) takes the following form:

$$\frac{\partial\overline{n}_\omega^{eff}(x,t)}{\partial x}\ll k_0\left[\overline{n}_\omega^{eff}(x,t)\right]^2 \tag{5a}$$

Relation (4) under the condition of (5a) is fulfilled, as a result of the series arrangement, it is simplified and will have the following form

$$\frac{\partial\phi}{\partial x}=k_0\overline{n}_\omega^{eff}(x,t)\sqrt{1+i\frac{1}{k_0^2\left[\overline{n}_\omega^{eff}(x,t)\right]^2}\frac{\partial\overline{n}_\omega^{eff}(x,t)}{\partial x}}\pm k_0\overline{n}_\omega^{eff}(x,t)+\frac{i}{2\overline{n}_c}\frac{\partial\overline{n}_\omega^{eff}(x,t)}{\partial x} \tag{5b}$$

We seek a partial solution of the wave equation (3) in the following (6) form

$$E_*^t(x,t)=A\frac{1}{\sqrt{\overline{n}_\omega^{eff}(x,t)}}\exp\left(\pm ik_0\int\overline{n}_\omega^{eff}(x,t)\,dx\right) \tag{6}$$

The complex amplitude of the magnetic field strength is obtained according to the wave equations [2] by differentiation $H_*^t(x,t)=-\frac{1}{i\omega\mu_0}\frac{\partial E_{*y}^t(x,t)}{\partial x}$.

In this H_*^t is the component of the magnetic field strength vector along the 0Z axis, and $\eta_0=\sqrt{\mu_0/\varepsilon_0}$ is the wave resistance (impedance) in vacuum. If the weak variation condition (5) is satisfied, we obtain

$$H_*^t(x,t)=\pm\frac{A}{\eta_0}\sqrt{\overline{n}_\omega^{eff}(x,t)\exp\left(\pm ik_0\int\overline{n}_\omega^{eff}(x,t)\,dx\right)} \tag{7}$$

It should be noted that the choice of sign in expressions (6) and (7) has more illustrative than physical meaning.

The condition of weak variability of the field (5a) in the coordinate space is clearly formulated if we take into account that

$$k_0\overline{n}_\omega^{eff}(x,t)=\frac{2\pi v_0}{c_0}\overline{n}_\omega^{eff}(x,t)=\frac{2\pi}{\lambda_0}\overline{n}_\omega^{eff}(x,t)=\frac{2\pi}{\lambda_\omega^{eff}(x,t)}=k_\omega^{eff}(x,t),$$

where $\lambda_\omega^{eff}(x,t)=\lambda_0/\overline{n}_\omega^{eff}(x,t)$ and $k_\omega^{eff}(x,t)=k_0\overline{n}_\omega^{eff}(x,t)$ (here $\lambda_0=c_0/v_0$ and $k_0=2\pi/\lambda_0$ is the electromagnetic wavelength and wave vector in vacuum) is the variable wavelength of the microwave EMF and the wave vector in a continuous porous (effective) medium. Then the corresponding relation (5) acquires a transparent physical meaning and the condition for it can be written:

$$\frac{1}{\overline{n}_\omega^{eff}(x,t)}\frac{\partial\overline{n}_\omega^{eff}(x,t)}{\partial x}\ll k_\omega^{eff}(x,t) \tag{8}$$

where $k_\omega^{eff}(x,t)=\omega_0/\overline{c}_\omega^{eff}(x,t)$ (here $\overline{c}_\omega^{eff}(x,t)=c_0/\overline{n}_\omega^{eff}(x,t)$ is the speed of light propagation in an effective medium) is the wave vector in a porous (wetted) medium modeled according to the effective dielectric characteristics [3].

The condition of weak variation of the dielectric properties of the material in the coordinate form (6) is usually ensured for most composite bodies with low electrical conductivity (2) due to the large value of the linear frequency ν_0 of microwave radiation in vacuum. In the following, we will neglect the dispersion of the wave characteristics of the material under study at a constant value $\omega_0 = 2\pi\nu_0$ of the angular frequency of microwave radiation in vacuum, considering the propagation of electromagnetic waves in the internal volume of a porous (wetted) flat plate, which takes place when taking into account scattering effects.

Thus, we arrive at a closed (complete) set of conditions for weak variability of dielectric (wave), bulk (phase) and wave properties of a three-phase (wetted) porous medium in the coordinate and time space of variables

$$\frac{1}{\theta_\sigma(x,t)}\frac{\partial\theta_\sigma(x,t)}{\partial t} \ll \omega_0$$

and

$$\frac{1}{\bar{n}_\omega^{eff}(x,t)}\frac{\partial\bar{n}_\omega^{eff}(x,t)}{\partial x} \ll k_\omega^{eff}(x,t) \quad (9)$$

$$\lambda_\omega^{eff} = \frac{2\pi\upsilon_\omega^{eff}(x,t)}{\omega_0} \gg l \quad (10)$$

Here $k_\omega^{eff}(x,t) = 2\pi/\lambda_\omega^{eff}(x,t)$ і $\upsilon_\omega^{eff}(x,t) = c_0/\bar{n}_\omega^{eff}(x,t)$ is the wave vector and phase velocity of electromagnetic (TEM) wave propagation in the simulated medium, is the effective value of the refractive index, $\bar{n}_\omega^{eff}(x,t)$ is the volume fraction $\sigma = \{S,L,G\}$ phase ω_0, - constant angular frequency of EMF in the microwave frequency range, l is the characteristic area size (REV) [1] for space averaging.

Relations (9)-(10) make it possible to apply the approximation of the effective macroscopic field [4] when studying (calculating) the effective electrophysical characteristics of a porous material within the framework of the approximation [1] of the theory of local spatial averaging.

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According to the properties of the solutions of the W.K.B model, when the conditions of weak variability (9) and (10) of the electrophysical characteristics of the porous (wetted) material are satisfied, the general solution of the wave equation (3) for a plane transverse (T.E.M.) electromagnetic wave can be written [14] in the form

$$E_*^t(x,t) = \frac{1}{\sqrt{\bar{n}_\omega^{eff}(x,t)}}\left(A(t)\sqrt{\bar{n}_\omega^L(t)}\,Exp\left[-ik_0\int_{-L/2}^{x}\bar{n}_\omega^{eff}(x,t)\,dx\right] + B(t)\sqrt{\bar{n}_\omega^R(t)\,Exp\left[ik_0\int_{L/2}^{x}\bar{n}_\omega^{eff}(x,t)\,dx\right]} \right) \quad (11)$$

where $\bar{n}_\omega^\alpha(t) \equiv \bar{n}'(\pm L/2,t), \alpha = \{L,R\}$ is the value of the refractive index that varies over time due to the sorption or desorption of liquid on the surfaces of the plate, $A(t)$ and $B(t)$ are unknown functions of time, and $\exp[\ldots]$ is notation of the exponent.

In the left (L) and right (R) half-space, external (Figure 1) with respect to the boundaries (edges) of the plate, which is modeled by vacuum, the electric field strength (under external microwave irradiation) will be considered in the form of expressions

$$E_*^t(x,t) = E_L\left[Exp\left[-ik_0(x+L/2)\right] + C(t)\,Exp\left[ik_0(x+L/2)\right]\right] \quad (12)$$

$$E_*^R(x,t) = E_R\left[D(t)\,Exp\left[-ik_0(x-L/2)\right] + Exp\left[ik_0(x-L/2)\right]\right] \quad (13)$$

where C(t) and D(t) are unknown functions, E_L and E_R are actual amplitudes of the external electromagnetic field (E.M.F.).

Let us write the boundary conditions for the components (Figure 1) of the electric field strength

$$E_{*y}^L(-L/2,t) = E_{*y}^t(-L/2),\; E_{*y}^L(L/2,t) = E_{*y}^t(L/2) \quad (14)$$

and derivatives that correspond to the component strengths of the magnetic field

$$\partial E_{*y}^L(-L/2,t)/\partial x = \partial E_{*y}^L(-L/2)/\partial x, \partial E_{*y}^R(L/2,t)/\partial x = \partial E_{*y}^t(L/2)/\partial x \quad (15)$$

Such conditions (14) and (15) make it possible to take into account the dependence of the field amplitudes on the wetting at the boundaries (edges) of the plate, the value of which (volume fraction θ_L of liquid or saturation η_L of pores with liquid) changes dynamically depending on the power of the internal (thermal) heating sources and the implementation of heat and mass transfer conditions at the edges of the plate.

In accordance with the described expressions (11), (12) and (13) for the electric field strengths according to the continuity condition (14) of the components

$$E_L\left[1+C(t)\right]=A(t)+B(t)\gamma_{RL}(t)\delta(t)E_R\left[D(t)+1\right]=A(t)\gamma_{LR}(t)\delta(t)+B(t) \quad (16)$$

and continuous (15) of derivatives

$$E_L\left[1-C(t)\right]= =\overline{n}_\omega^L(t)\left[A(t)-B(t)\gamma_{RL}(t)\delta(t)\right]E_R\left[D(t)-1\right]= =\overline{n}_\omega^R(t)\left[A(t)\gamma_{LR}(t)\delta(t)-B(t)\right] \quad (17)$$

we obtain a system of linear equations (16) and (17) for determining the unknown (variable in the simulation time interval) coefficients that specify the amplitude of the E.M.F. at the edges of the plate, in the accepted notations

$$\gamma_{LR}(t)=\sqrt{\frac{\overline{n}_\omega^R(t)}{\overline{n}_\omega^L(t)}}=\frac{1}{\gamma_{RL}(t)} \text{ and } \delta(t)=\exp\left[-ik_0\int_{-L/2}^{L/2}\overline{n}_\omega^{eff}(x,t)dx\right] \quad (18)$$

Such coefficients, taking into account the relation (18), will have the form

$$A(t)=E_L\frac{T_L^t-(E_R/E_L)\delta(t)T_R^tR_L^t\gamma_{RL}(t)}{1-\left[\delta(t)\right]^2R_L^tR_R^t} \text{ and } B(t)=E_R\frac{T_R^t-(E_L/E_R)\delta(t)T_L^tR_R^t\gamma_{LR}(t)}{1-\left[\delta(t)\right]^2R_L^tR_R^t} \quad (19)$$

where $T_\alpha^t=2/\left[1+\overline{n}_\omega^\alpha(t)\right]$ i $R_\alpha^t=\left[1-\overline{n}_\omega^\alpha(t)\right]/\left[1+\overline{n}_\omega^\alpha(t)\right]$ (here $\alpha=\{L,R\}$ is the index corresponding to the left (*L*) and right (*R*) edges of the plate) are, respectively, the transmission and reflection coefficients of the T.E.M wave at the plate boundaries.

Thus, according to (11) and the expressions for the coefficients and (19) which are determined from the continuity conditions (14) and (15) of the field components, we obtain a partial solution of the wave equation (3) in the electric and magnetic field strengths

$$E_{*y}^L(x,t)=\frac{1}{\sqrt{\overline{n}_\omega^{'}(x,t)}}\left(\overline{A}(t)Exp\left[-ik_0\int_{-L/2}^{x}\overline{n}_\omega^{eff}(x,t)dx\right]+\overline{B}(t)Exp\left[-ik_0\int_{x}^{L/2}\overline{n}_\omega^{eff}(x,t)dx\right]\right) \quad (20)$$

$$H_{*z}^L(x,t)=\frac{1}{\eta_0}\sqrt{\overline{n}_\omega^{'}(x,t)}\left(\overline{A}(t)Exp\left[-ik_0\int_{-L/2}^{x}\overline{n}_\omega^{eff}(x,t)dx\right]-\overline{B}(t)Exp\left[ik_0\int_{x}^{L/2}\overline{n}_\omega^{eff}(x,t)dx\right]\right) \quad (21)$$

here $\overline{A}(t)=A(t)\sqrt{\overline{n}_\omega^L(t)}$ i $\overline{B}(t)=B(t)\sqrt{\overline{n}_\omega^R(t)}$ - determined according to relations (19) and normalized according to the values $\overline{n}_\omega^\alpha(t)=\overline{n}_\omega^{eff}(\pm L/2,t)$ (where $\alpha=\{L,R\}$ - index of the plate edges) of the refractive index at the edges of the plate complex amplitudes of the electromagnetic wave, and $\eta_0=\sqrt{\mu_0\varepsilon_0}$ - characteristic resistance (impedance) in a vacuum.

It is known [7] that for the wave vector of a T.E.M. wave in a continuous (effective) medium the following relation holds: $k_0\overline{n}_\omega^{eff}(x,t)=\beta(x,t)-i\alpha(x,t)$, where $\alpha(x,t)=-k_0\,\mathrm{Im}\left[\sqrt{\overline{n}_\omega^{eff}(x,t)}\right]$ and $\beta(x,t)=k\ \mathrm{Re}\left[\sqrt{\overline{n}_\omega^{eff}(x,t)}\right]$ are absorption and propagation indicators of T.E.M waves.

Let us define the tangent of the dielectric loss angle $tg\delta_{\ddot{u}}$ in the simulated porous (wetted) material

$$tg\delta_\omega = -\frac{\mathrm{Im}\left[\bar{\varepsilon}_\omega^{eff}\right]}{\mathrm{Re}\left[\bar{\varepsilon}_\omega^{eff}\right]} = \bar{\varepsilon}_\omega^{eff(2)} / \bar{\varepsilon}_\omega^{eff(1)} \quad (22)$$

where $\bar{\varepsilon}_\omega^{eff} = \bar{\varepsilon}_\omega^{eff(1)} - \bar{\varepsilon}_\omega^{eff(2)}$ is the effective generalized complex dynamic permittivity (E.G.C.D.D.P), and $\bar{\varepsilon}_\omega^{eff(1)} = \mathrm{Re}\left[\bar{\varepsilon}_\omega^{eff}\right]$ i $\bar{\varepsilon}_\omega^{eff(2)} = \mathrm{Im}\left[\bar{\varepsilon}_\omega^{eff}\right]$ denoting the real and imaginary parts respectively.

Then the absorption $\alpha(x,t)$ and propagation $\beta(x,t)$ indices of the T.E.M wave are described by the expressions

$$\alpha(x,t) = k_0 \frac{\sqrt{\mathrm{Re}\left[\bar{\varepsilon}_\omega^{eff}(x,t)\right]}}{\sqrt{2}} \sqrt{\sqrt{1+tg^2\delta_\omega(x,t)}+1} \quad (23)$$

$$\beta(x,t) = k_0 \frac{\sqrt{\mathrm{Im}\left[\bar{\varepsilon}_\omega^{eff}(x,t)\right]}}{\sqrt{2}} \sqrt{\sqrt{1+tg^2\delta_\omega(x,t)}-1} \quad (24)$$

subject to performance $2\alpha\beta = k_0^2 \sqrt[3]{tg^2\delta_\omega}\,\mathrm{Re}\left[\bar{\varepsilon}_\omega^{eff}\right]$ normalizing ratio.

Considering the above ratio $k_0\bar{n}_\omega^{eff}(x,t) = \beta(x,t) - i\alpha(x,t)$, where $\alpha(x,t) = -k_0\,\mathrm{Im}\left[\sqrt{\bar{n}_\omega^{eff}(x,t)}\right]$ and $\beta(x,t) = -k_0\,\mathrm{Re}\left[\sqrt{\bar{n}_\omega^{eff}(x,t)}\right]$ are the above-mentioned absorption and propagation parameters of a plane electromagnetic wave, we obtain the effective values of the wavelength $\lambda_\omega^{eff}(x,t)$ and the depth $D_p^{eff}(x,t)$ of wave penetration

$$\lambda_\omega^{eff}(x,t) = \lambda_0 \frac{\sqrt{2}}{\sqrt{\bar{\varepsilon}_\omega^{eff(1)}}} \frac{1}{\sqrt{\sqrt{1+tg^2\delta_\omega(x,t)}+1}}$$

$$D_p^{eff}(x,t) = \lambda_0 \frac{c_0}{\omega} \frac{\sqrt{2}}{\sqrt{\bar{\varepsilon}_\omega^{eff(2)}}} \frac{1}{\sqrt{\sqrt{1+tg^2\delta_\omega(x,t)}-1}},$$

where $tg\delta_\omega(x,t) = \bar{\varepsilon}_\omega^{eff(2)} / \bar{\varepsilon}_\omega^{eff(1)}$ is the dielectric loss tangent.

The power of the E.M.F energy flow in a diffuse wave process can be calculated according to the definition of the Umov-Pointing vector [4] by the relation

$$P_x^t(x) = \frac{1}{2}\mathrm{Re}\left[E_{*y}^t(x)\tilde{H}_{*z}^t(x)\right] \quad (25)$$

where $\tilde{H}_{*z}^t \equiv H_{*z}^t$ is the complex conjugate value of the averaged amplitude of the magnetic field strength.

Analyzing expression (18), for the model parameter we obtain

$$\delta(t) = \exp\left[-ik_0 \int_{-L/2}^{L/2} \bar{n}_\omega^{eff}(x,t)\right] = e^{-\bar{\alpha}(t)}\left[\cos\left[\bar{\beta}(t)\right] - i\sin\left[\bar{\beta}(t)\right]\right] = \bar{\delta}(t)e^{-i\bar{\beta}(t)} \quad (26)$$

here i $\bar{\alpha}(t) = -\int_{-L/2}^{L/2}\alpha(x,t)dx$ are integral values of the absorption and propagation coefficients of an electromagnetic wave through the thickness L of plate correspondingly, and $\bar{\delta}(t) = e^{-\bar{\alpha}(t)}$ is the amplitude multiplier, which is determined by the damping coefficient that varies with time (due to the movement of fluid phases in a porous material).

Then, according to the definition (25) and the expressions for the electric (20) and magnetic (21) field strengths for the microwave radiation flux power in the accepted (26) notations in the large thickness approximation L of plate $\left(\bar{\delta}(t) \simeq 0\right)$ (26) and symmetrical edges ($E_L = E_R = E_0$) (if E_0 - amplitude value of the external E.M.F intensity), the relationship for the power of the energy flow in a porous wetted material with effective wave properties will be

$$P_x^t(x) = P_{in}\,|\bar{n}_w(t)|\,|T(t)|^2 \sqrt{\frac{1}{2}\left(1+\frac{1}{\sqrt{1+tg^2\delta_\omega(x,t)}}\right)} \times G_L^t(\alpha(x),L) \quad (28)$$

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Were,

$$G_L^t\left(\alpha(x),L\right)=\exp\left[-2\int_{-L/2}^{x}\alpha(x,t)\,dx\right]+$$
$$+\bar{\delta}(t)e^{-i\bar{\beta}(t)}\left\{\bar{\tilde{A}}(t)\bar{B}(t)Exp\left[2i\int_{-L/2}^{x}\beta(x,t)\,dx\right]-\bar{A}(t)\bar{\tilde{B}}(t)Exp\left[2i\int_{x}^{L/2}\beta(x,t)\,dx\right]\right\}+$$
$$+\exp\left[-2\int_{x}^{L/2}\alpha(x,t)\,dx\right]$$

is the notation; $P_{in}=E_0/(2\eta_0)$ is the power of external microwave radiation; $\bar{n}_{\omega}(t)=\bar{n}_{\omega}^{eff}(\pm L/2,t)$ and $T(t)=2/\left[1+\bar{n}_{\omega}(t)\right]$ is the the value of the refractive index and transmission coefficient of the electromagnetic T.E.M. wave at the edges of the plate; $\alpha(x,t)=-k_0\,\mathrm{Im}\left[\sqrt{\bar{n}_{\omega}^{eff}(x,t)}\right]$ is the attenuation coefficient of the T.E.M. wave.

Let us assume that the total amount of E.M.F energy supplied to the material under study at microwave intermediate resolution is completely converted into heat due to scattering (diation) effects. Thus, we neglect the processes of performing work by the studied thermodynamic system due to internal (ponderomotor) forces acting on the porous frame (including thermal expansion of the plate).

Then the expression for calculating the power of electromagnetic (dielectric or microwave) heating sources has the form

$$\dot{q}^t(x)=-\partial P_x^t(x)/\partial x \quad (28)$$

where $P_x^t(x)$ is the value of the Umov-Pointing vector [4] at the edges and in the internal volume of the plate.

For the power of internal heat sources $\dot{q}^t(x)$ under symmetric external (microwave) irradiation, which is determined by the expression for the E.M.F energy flow in the material (25), when approximating large thicknesses ($\bar{\delta}(t)\simeq 0$) of the plate according to (26), we obtain

$$\dot{q}^t(x)=2P_{in}\bar{n}_{\omega}(t)\left(T(t)\right)^2\alpha(x,t)\sqrt{\frac{1}{2}\left(1+\frac{1}{\sqrt{1+tg^2\delta_{\omega}(x,t)}}\right)}\times G_L^t\left(\alpha(x),L\right) \quad (29)$$

In relation (29), the notations are equivalent to the physical quantities described according to (28) and

$$G_L^t\left(\alpha(x),L\right)=Exp\left[-2\int_{-L/2}^{x}\alpha(x,t)\,dx\right]+Exp\left[-2\int_{x}^{L/2}\alpha(x,t)\,dx\right]$$

The numerical modelling

As before, the Bruggeman formula [3] was used in the numerical simulation of the heat source power (28) in the EMA approximation.

$$\sum_{\sigma}\theta_{\sigma}\frac{\varepsilon_c^{\sigma}(\omega)-\bar{\varepsilon}_{\omega}^{eff}}{\varepsilon_c^{\sigma}(\omega)+2\bar{\varepsilon}_{\omega}^{eff}}=0 \quad (30)$$

and the expression (31) obtained in [15] for finding the uniform wetting η_L^{eqv} of a porous material

$$\eta_L^{eqv}=\left(\eta_L^{cr}-\eta_L^{ir}\right)/\left[1+\left(\frac{P_{amb}\sqrt{k_{in}/\varphi}}{\sigma}\right)^{\frac{1}{1-m}}\right]^m+\eta_L^{ir} \quad (31)$$

here φ - averaged porosity, η_L^{cr} and η_L^{ir} - critical and residual pore saturation with liquid, P_{amb} - pressure value in the surrounding environment, k_{in} - inherent permeability of the frame, σ - surface tension coefficient, m - model parameter.

For the material under study, namely historical ceramic brick ($\varphi=0,46$ - averaged porosity) and two modification ($\varphi=0,4$ - modA and $\varphi=0,52$ - modB), with known values of dielectric (static) permittivities of phases,

Table 1: Dielectric constants of components for the studied material (historical ceramic brick) according to the spherical inclusion model [ε_S - solid phase, ε_L - liquid phase (water), ε_G - gas phase (dry air)].

ν, Gz	ε_S		ε_L		ε_G	
	ε'_S	ε''_S	ε'_S	ε''_L	ε'_G	ε''_G
2.45x10⁹	5,86	0,703	80	20	1	0

which are given above in Table 1, by relation (27) we obtain the corresponding graphical dependencies for the real $\overline{\varepsilon}_{\omega}^{eff(1)}$ (Figure 2) and imaginary $\overline{\varepsilon}_{\omega}^{eff(2)}$ (Figure 3) part of effective generalized complex dynamic permittivity (EUCDDP) $\overline{\varepsilon}_{\omega}^{eff}$ porous moistened material at a typical (standardized) frequency of external microwave irradiation $v_0 = 2.45 \cdot 10^{-9} [1/s]$ ($2,45MGz$) from the saturation η_L by liquid.

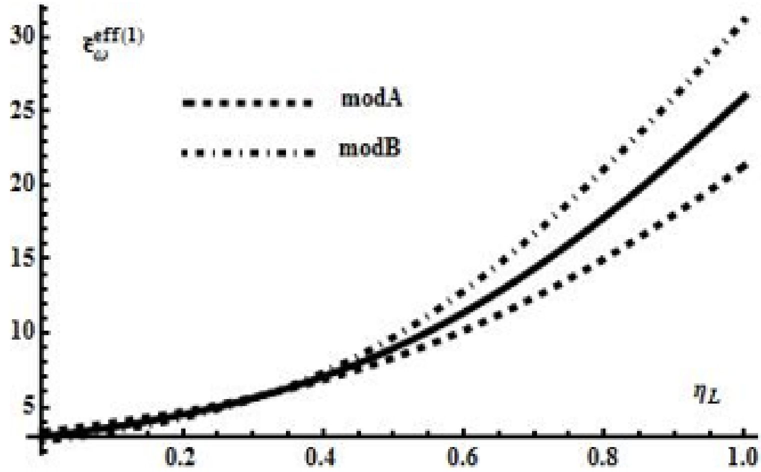

Figure 2: Real part of E.G.C.D.D.P. $\vec{a}_{u}^{eff}$ for the material under study, as a function of pore saturation by liquid.

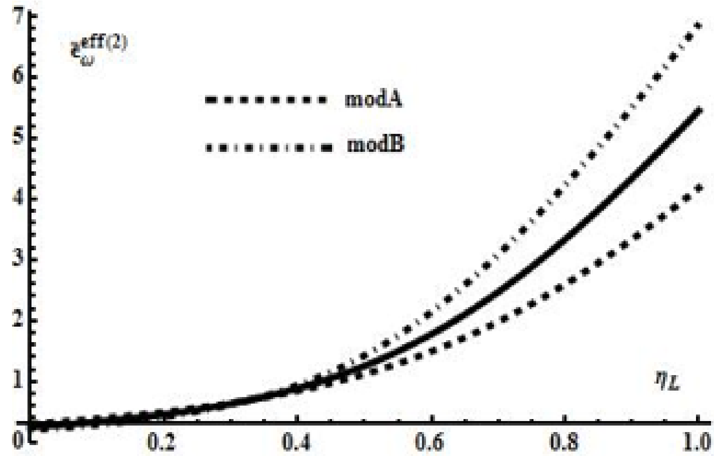

Figure 3: Imaginary part of E.G.C.D.D.P $\vec{a}_{u}^{eff}$ for the material under study, as a function of pore saturation η_L by liquid.

Below is also graphically depicted (Figure 4) the dependence of the dielectric loss tangent angle $tg\delta_{\omega} = \overline{\varepsilon}_{\omega}^{eff(2)} / \overline{\varepsilon}_{\omega}^{eff(1)}$ at different humidity values η_L of the investigated porous material at a specified frequency of microwave irradiation.

For such a material, according to the moisture dependence η_L from the effective pore radius $\overline{r}$ is established [15], that under porosity $\varphi = 0,46$ from the effective pore radius $\eta_{L}^{ir} = 0,015$ and critical $\eta_{L}^{cr} = 1$ pore fluid saturation, as well as the value of the model parameter $m = 0,67$ in the semi-empirical model of moisture release or retention by van Genuchten [16]. Then, according to relation (31), the macroscopic (homogeneous or uniform) value of wetting of such (porous) material under normal conditions is (when $P_{G}^{amb} = 101325[Pa]$, where P_{G}^{amb} - external pressure relative to the moistened macroscopic porous material) corresponds to value $\eta_{L}^{eqv} = 0,64$, which coincides with the maximum of the pore size distribution function over the effective radius.

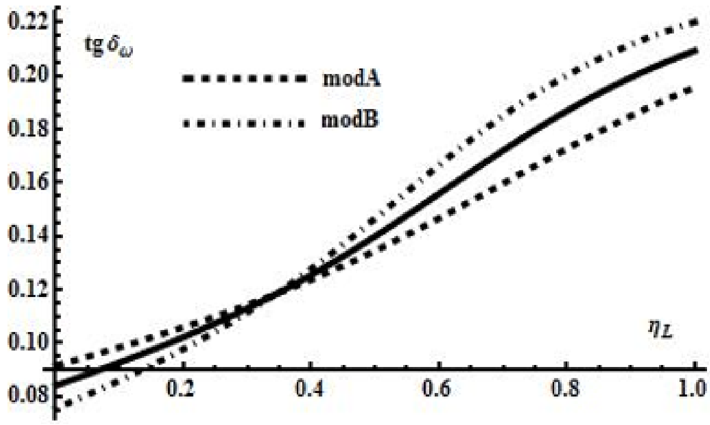

Figure 4: Dielectric loss tangent, as a function of pore saturation by liquid.

Under the described conditions of uniform (homogeneous) wetting of the porous material, the absorption indices (23) and propagation (24) of the TEM wave, as well as the transmission and reflection coefficients in expressions (1.16) for the amplitudes of the electromagnetic wave on the boundary surfaces of the porous plate (Figure 1) do not depend on time due to the constant value η_L^{eqv} of liquid saturation. In this case, the distribution of EMF power in the material under symmetric microwave irradiation according to (27) is determined by the thickness L of plate, equilibrium value of pore saturation with liquid η_L^{eqv} and dielectric (relaxation) properties of the phases. This is due to the fact that, that ЕУКДДП $\overline{\varepsilon}_\omega^{eff}$ moistened porous material ($\varphi = 0,46$)is defined according to the EMA approximation [3] (30) (Figure 2 and Figure 3), as well as the dielectric loss tangent $tg\delta_\omega$ (Figure 4), are determined by the equilibrium constant value η_L^{eqv} saturation of pores with liquid.

Then the power of internal heat sources (microwave heating) according to expression (28) for the EMF energy flow in the plate material can be calculated numerically and displayed graphically in Figure 5 and Figure 6.

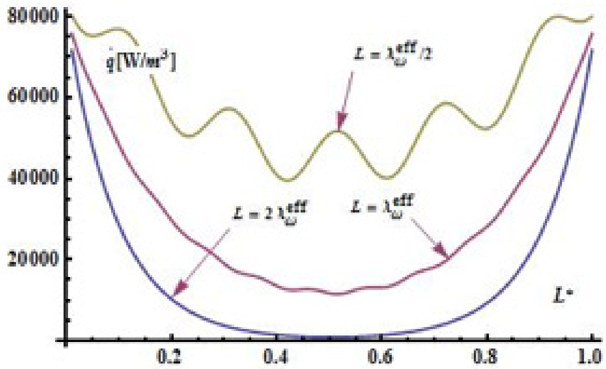

Figure 5: Power distribution of internal heating sources at large plate thicknesses.

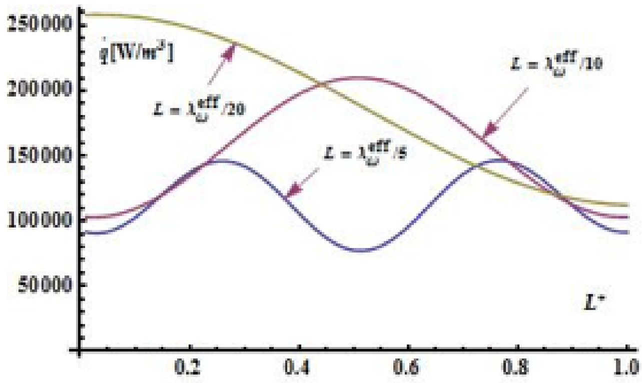

Figure 6: Power distribution of internal heating sources at small plate thicknesses.

Graphical results of numerical modeling (Figure 5 and Figure 6) of the power of internal (volumetric) heat sources $\dot{q}$ in a porous (unconfined) moistened plate at different thicknesses L of plate (where $L = n\Delta L / L\left(n = \Delta L / L\right)$ - dimensionless plate thickness, and n - number of breakpoints), which are multiples of the wavelength λ_{ω}^{eff} in the environment, obtained for the following values of microwave irradiation parameters and properties (parameters or characteristics) of the investigated porous material: 1. External microwave irradiation: power - $P_{in} = 2500\left[W / m^2\right]$, frequency - $\nu_0 = 2.45 \cdot 10^{-9}\left[1 / s^{-1}\right]$; 2. Porous humidified medium: $\varphi = 0,46$ - averaged porosity, $k_{in} = 2,29 \cdot 10^{-13}\left[m^2\right]$ - intrinsic permeability of the skeleton, $\eta_L^{eqv} = 0,64$ - equilibrium saturation of pores with liquid, $T = 298,15\left[K\right]$ - thermodynamic temperature, $P_G = 101325\left[Pa\right]$ - pressure value in the gas phase, $\lambda_{\omega}^{eff} = c_0 / \overline{n}_{\omega}^{eff} = 0,14\left[m\right]$ - the length of an electromagnetic wave in a medium.

The strength criterion for a moistened porous plate, the dielectric properties of which are described by relations (1)-(31), is formulated as [18]:

$$\sigma_0 - C_0 + C_1 P\left(J\right)\tau_0 + C_2\left(\tau_0\right)^2 = 0 \quad (32)$$

where $P(J)$ – function of the boundary surface in the deviatoric plane; $\sigma_{0=}I_1/3$ – average tension; $\tau_{0=(2J2/3)}^{0,5}$ – octahedral shear stress; I_1 the first invariant of the stress tensor; J_2, J_3 – second and third invariant of the stress deviator; J=3$\sqrt{3}$ $J_{3/(2(J2}$)^{1,5}) – alternative invariants of stress deviators J_2, J_3; α, β, C_0, C_1, C_2 – material constants that are established experimentally based on the methodology [18].

There is a transition layer with sufficient adhesion between the liquid in the pore and the solid phase. Adhesive bond energy γ_{ad} and it changes $\Delta\gamma_{ad}$ depend on the energy characteristics of the coating and the porous carcase [19]. Let us formulate the strength criterion [19] for the transition layer at the pore boundary in the form

$$\Delta\sigma_m \le \Delta\sigma_{m*}, \quad \Delta\gamma_m \le \Delta\gamma_{m*}, \quad \Delta A_{ad} \le \Delta A_{ad*},$$

$$\Delta\gamma_{vad} \le \Delta\gamma_{ad*}, \quad (33)$$

where $\Delta\sigma_m$ – change in interfacial tension; $\Delta\gamma_m$ – change in interfacial energy; Δ – symbol of deviation (change) of a parameter or energy characteristic of the surface (interfacial) layer; ΔA_{ad} – change in adhesion work; $\Delta\sigma_{m\#}$, $\Delta\gamma_{m\#}$, $\Delta A_{ad\#}$, $\Delta\gamma_{ad\#}$– empirical constants.

Relations (31), (32) together constitute a generalized version of the strength criterion for a porous material (such as concrete). The parameters of expression (31) are determined on the basis of experiment, and the parameters of relations (32) are estimated on the basis of a computational experiment [20].

If adhesion is broken, the pore size increases, the pores merge and can form a crack (a microcrack to begin with).

The criteria for ultimate plasticity Θ_T and strength Θ_S for the tip of a defect (microcrack) can be formulated as dimensionless relations. Критерії граничних пластичності та міцності для вершини дефекту (мікротріщини) можна сформулювати у вигляді безрозмірних співвідношень [18]:

$$\Theta_T = \frac{\varepsilon_i}{\varepsilon_{ic}} + \frac{\varepsilon_0}{\varepsilon_{0c}} \quad (34)$$

$$\Theta_s = \frac{\varepsilon_i \cdot \cos\phi_r}{\varepsilon_{iu}} + \frac{\varepsilon_0}{\varepsilon_{0u}}, \quad (35)$$

where ε_i - intensity of deformations; ε_0 - volumetric deformation; ϕ_r - volumetric deformation, рад; ε_{ic} - intensity of destructive deformations; ε_{0c} - destructive volumetric deformation; ε_{iu} – ultimate strain intensity; ε_{0u} -limiting volumetric deformation.

Transition from deformation components ε_{jk} (34) and (35) to mechanical stresses σ_{jk} We carry out the calculations based on the model relations described in the work [19] (*j*, *k* =1, 2, 3). In particular, an equation of the Hooke's law type is used.

Due to the influence of internal gas and liquid pressure in the pore, a zone of plastic deformation appears in the vicinity of the crack tip, to which the pressure corresponds $p_{cr1}(\sigma_T)$, and in the second stage – a critical state $p_{cr2}(\sigma_S)$ [σ_T – plastic limit; σ_S – strength limit (destructive stress of a solid body, i.e. a skeleton)]

Parameters p_{cr1} and p_{cr2} are important for assessing the strength, reliability, and service life of a porous material under the influence of internal (volumetric) heat sources.

Let us supplement the relation (32)-(35) with the expression of the limitation on the power of internal heat sources $\dot{q}^t(x)$, which is determined from the relation (29) taking into account (30) and (31):

$$\dot{q}^t(x) \le \dot{q}_*(x) \quad (36)$$

where $\dot{q}^t(x)$ – the maximum value of the power of internal heat sources that lead to the destruction of the porous body.

Relations (29)–(36) constitute a new version of the strength criterion for a porous body (compared to (32)–(35)) taking into account the power of internal heat sources that lead to the destruction of the porous body (i.e., to the formation of a micro crack, which over time can turn into a main crack).

Conclusions

From the considered graphical dependencies it follows that at plate thicknesses the exponential law of power attenuation of Beer-Lambert microwave heating sources is realized [16]. If $L \le \lambda_\omega^{eff}$ wave a periodical effects occur (see Figure 5 (when $L = \lambda_\omega^{eff}$ and $L = \lambda_\omega^{eff}/2$) and Figure 6 (when $L = \lambda_\omega^{eff}/5$ and $L = \lambda_\omega^{eff}/10$), which can be localized at plate thicknesses that are commensurate with the minimum (Fig. 6,when $L = \lambda_\omega^{eff}/20$ and $L = \lambda_\omega^{eff}/10$) symmetric with respect to the center of the plate, the anharmonicity value. It is clear that the wave (aperiodic) properties of heat sources will change depending on the value of equilibrium humidification η_L^{eqv} porous plate material, which is determined by the structure parameters: φ - porosity and k_{in} - the inherent permeability of the skeleton. Criterion ratios have been formulated for assessing the strength of a porous material, taking into account the power of internal heat sources that lead to the destruction of a porous body.

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Conflict of interest

No conflict of interest.

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