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Understanding The Structure and Parameters Identification of Compliant Robot Arms Using Frequency Domain Analysis

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The following article investigates the dynamic performance of robotic arms used in various manipulators within the modern automation industry. The minimum of point-to-point settling time is considered by low damping of the arms' vibrations. The compliance of linkage/ transmission connecting the load to the motor manifests such vibrations (resonance phenomena). The fundamental but most complex part of affective control design is selecting the appropriate the model of electromechanical structure of robotic arm for subsequent its parameters identification. The paper presents the method for analyzing at experimental studies in the frequency domain. The resonance is considerate as well-known typical compliant models of compliant reducers. Provided is the calculation of the SCARA robot's arm parameters. The procedure can be used for models with three and more bodies which covers most practical scenarios. The identification procedure enables the detecting the locations of key compliant couplings. Therefore, we point out ways to enhance the stiffness for possibility increasing servo bandwidth of controlled robot axis and, consequently, the overall performance of the robot itself.

Keywords: Robot Arm; Frequency Domain; Bode Plots; Compliant Reducer; Resonance, Stiffness**Introduction**

The robot arms based on rotary electric brushless motors are widely used for various robot configurations – horizontal (SCARA) or vertical (PUMA) configurations. This is thanks to its high dynamics owing low moments of inertia and high energy efficiency compared to any other type of electric motors. The dynamic performance of the robot axes is different from ones of other servo

systems namely in the lower range of first resonance (Figure 1). Further its improvement is limited by the mechanical resonance. Proper control design of such axes is based on an identification procedure from choosing a model of an electromechanical structure with further parameters identification. We must point out that choosing a model is one of the more important and complex tasks of robots' arms design.

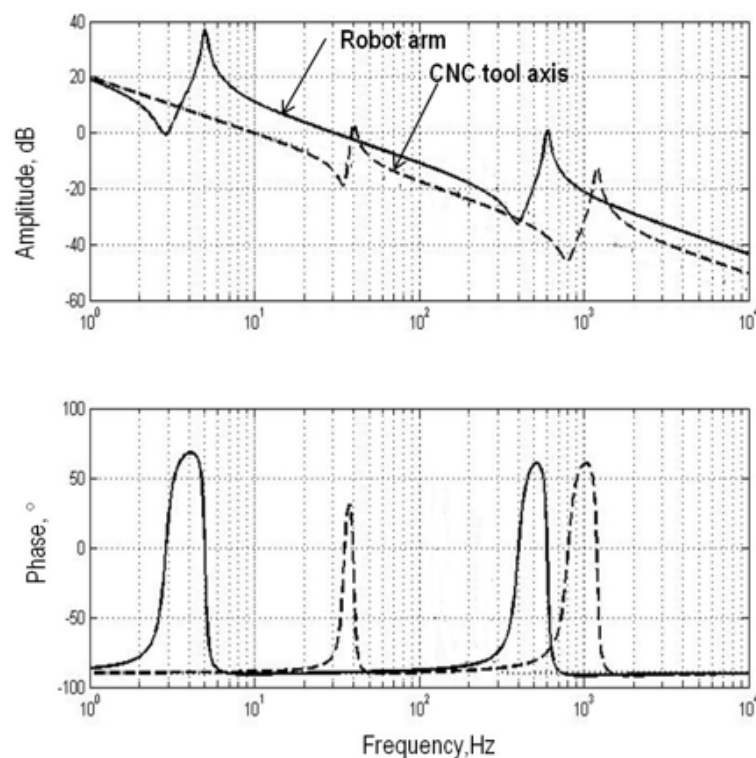


Figure 1: Bode of two industrial axes.

The existence of resonances due to compliant linkages is represented by pick-ups and pick-downs for each resonance (Figure 1). It is considered that pick-downs are Anti-Resonance (AR), and pick-up frequencies are Resonance (R) frequencies.

The pairs of AR and R frequencies represent a well-known compliant reducer type of resonance [1]. The Bode plots of a compliant reducer have key features [2] namely

- AR frequency is lower than R frequency.
- Bode phase increasing up to $(+180^\circ)$ between AR and R frequencies.
- AR frequencies are ones with “rigid” attachment of motor.
- Mismatch ratio (ratio of bodies’ moment of inertia) of each resonance is defined by the second power of R to AR frequency ratios.
- the Bode amplitudes increases after each resonance where amplitude increment L is defined by the amount of conditional switched off body moment of inertia.

Designing the axis’ model allows us to connect the model parameters with linear axis parameters which also help us understand the physical processes clearly. All well-known identification techniques are based on the analysis in time-domain [3] or frequency-domain [4-5]. Despite the existence of theoretic

equivalence between both approaches [6], the frequency-domain one has practical advantages especially when using multi-resonance compliant axes. Furthermore, built-in identification utility of modern motion controllers (ACS, Elmo, etc.) based on frequency-domain analyses, are more familiar for motion control engineers [7]. Such tools - along with the wide bandwidth of modern PWM current amplifiers (at least 1 - 2 kHz) - allow detecting resonance phenomena by experimental Bode plots accurately under environmental conditions.

The resonance phenomena of rotary actuators are described by a well-known two-body compliant model a.k.a. compliant reducer [1,7-8]. This model can be used for the identification of linear stage parameters [9]. In most practical case studies resonances are apart from [10], which allows coupling ones. The main motive is to detect the locations of the lowest compliant couplings that allow us to increase stiffness and thus improve the performance of robot arms. In this article, an example of experimental Bode plots of SCARA robot arm (“shoulder”) is presented. Further describing ones by approximate transfer functions allows us to choose the appropriate model based on the connected compliant reducer resonance model. The parameters identification procedure based on experimental data is presented as a “step-by-step” identification algorithm. The method to implement the algorithm for multi-resonance (2 resonances and more) axes will also be suggested.

The model block diagram design

Based on the above considerations, the robot arm model may be represented by a mathematical model with three bodies, where

viscous friction between masses is neglected for simplicity (Figure 2). The model is based on the two-body compliant reducer model in which third body is connected using the same principle as the second body of the compliant reducer.

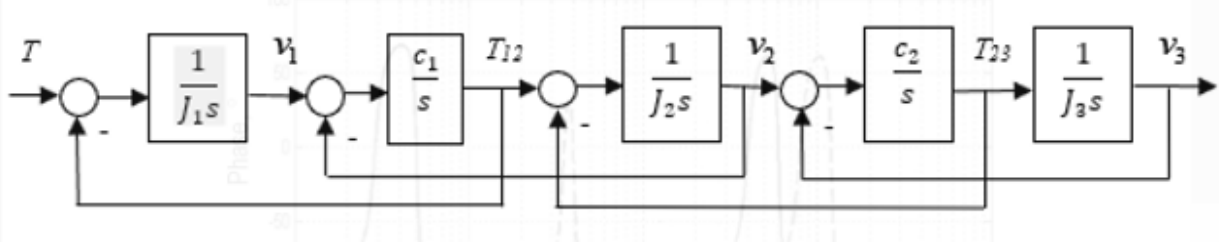


Figure 2: Three bodies model block diagram.

As shown in Figure 2:

- $V_1(s), V_2(s), V_3(s)$ - angular speeds of first (motor), second and third bodies,
- $T(s)$ - motor's torque,
- $T_{12}(s), T_{23}(s)$ - compliant torques between bodies,
- $J_1(s), J_2(s), J_3(s)$ - moments of inertia of first, second, and third

bodies,

- C_1, C_2 - coupling's stiffness between bodies.

The transfer function $H_A^*(s)$ of the motor's speed, when each resonance will be present separately, is best describes the experimental Bode as two serially connected compliant reducers [1]

$$H_A(s) = \frac{v_1(s)}{T(s)} = \frac{1}{J \cdot s} \frac{(P_{AR1}^2 s^2 + P_{D1} s + 1)}{(P_{R1}^2 s^2 + P_{D1} s + 1)} \frac{(P_{AR2}^2 + P_{D2} s + 1)}{(P_{R2}^2 + P_{D2} s + 1)} \quad (1)$$

In equation (1):

- J - total moment of inertia.
- $P_{AR1}=1/(2\pi \cdot f_{AR1}), P_{AR2}=1/(2\pi \cdot f_{AR2})$ - the periods of AR frequencies.
- $P_{R1}=1/(2\pi \cdot f_{R1}), P_{R2}=1/(2\pi \cdot f_{R2})$ - the periods of R frequencies.
- P_{D1}, P_{D2} - viscous friction (damping) time constant of first and second resonances.

Both resonances are presented as the separate multipliers, where the numerator represents the AR-frequencies and the denominator - the R- frequencies. The lower frequency of first resonance is represented by the first and second *stiff coupled* bodies (J_1+J_2) that are compliantly linked with stiffness c_2 to the third body with moment of inertia J_3 (Figure 3a). The higher frequency of second resonance is represented by the first and second body moments of inertia only linked with stiffness c_1 while the third body is *switched off* (Figure 3b).

$$\begin{bmatrix} \dot{v}_3 \\ \dot{T}_{23} \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} 0 & J_3^{-1} & 0 \\ -c_2 & 0 & c_2 \\ 0 & (J_1 + J_2)^{-1} & 0 \end{bmatrix} \cdot \begin{bmatrix} v_3 \\ T_{23} \\ v_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ (J_1 + J_2)^{-1} \end{bmatrix} \cdot T; \quad (2)$$

for the second resonance:

$$\begin{bmatrix} \dot{v}_2 \\ \dot{T}_{12} \\ \dot{v}_1 \end{bmatrix} = \begin{bmatrix} 0 & J_2^{-1} & 0 \\ -c_1 & 0 & c_1 \\ 0 & -J_1^{-1} & 0 \end{bmatrix} \cdot \begin{bmatrix} v_2 \\ T_{12} \\ v_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ J_1^{-1} \end{bmatrix} \cdot T. \quad (3)$$

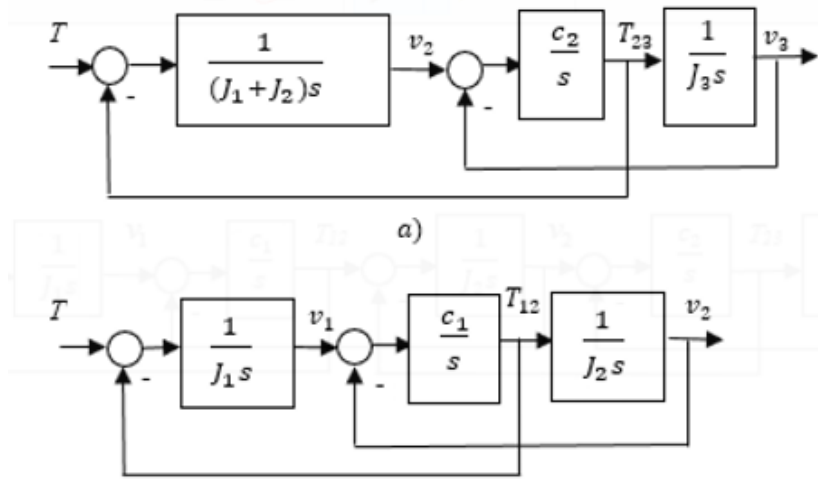


Figure 3: Block diagram models for first (a) and second (b) resonances.

Characteristic polynomials of both models are:

$$\det(sI - A_1) = s \left[s^2 + c_2(J_1 + J_2)^{-1} + c_2J_3^{-1} \right] \quad (4)$$

$$\det(sI - A_2) = s \left(s^2 + c_1J_1^{-1} + c_2J_2^{-1} \right) \quad (5)$$

where A_1 and A_2 matrixes are related to the first and second resonance models accordantly.

The transfer function $H_1(s)$ of the motor speed in general form

is:

$$H_1(s) = \frac{C_3 \cdot \text{Adj}(sI - A) \cdot B}{\det(sI - A)} \quad (6)$$

The $C_3 = [0 \ 0 \ 1]$ is output matrix for variable v_1 , where the transported state matrix for both cases are $x_1^T = [v_3 \ T_{23} \ v_2]$ and $x_2^T = [v_2 \ T_{12} \ v_1]$.

Therefore, the transfer functions $H(s)$ for both resonances are:

$$H_{1-1}(s) = \frac{(s^2 + c_1J_3^{-1}) \cdot (J_1 + J_2)^{-1}}{\det(sI - A_1)} = \frac{1}{Js} \cdot \frac{(J_3 \cdot c_2^{-1}s^2 + 1)}{\frac{(J_1 + J_2) \cdot J_3}{J_1 + J_2 + J_3} c_2^{-1}s^2 + 1}, \quad (7)$$

$$H_{1-2}(s) = \frac{(s^2 + c_1J_2^{-1}) \cdot J_1^{-1}}{\det(sI - A_2)} = \frac{1}{(J_1 + J_2)s} \cdot \frac{(J_2 \cdot c_1^{-1}s^2 + 1)}{\frac{(J_1 \cdot J_2)}{J_1 + J_2} \cdot c_1^{-1}s^2 + 1}. \quad (8)$$

Comparing expressions (7), (8) with (1), a connection between model parameters – the moments of inertia J_1, J_2, J_3 and the couplings stiffness c_1, c_2 to the AR and R resonance's periods are:

$$P_{AR1}^2 = J_3 \cdot c_2^{-1}; P_{R1}^2 = (J_1 + J_2) \cdot J_3 \cdot c_2^{-1} / (J_1 + J_2 + J_3), \quad (9)$$

$$P_{AR2}^2 = J_2 \cdot c_1^{-1}; P_{R2}^2 = J_1 \cdot J_2 \cdot c_1^{-1} / (J_1 + J_2). \quad (10)$$

The asymptotical Bode plots for the first body of the model (Figure 2) are presented on Figure 4. The single unit of Bode amplitude slope is equal to 20dB/decade slope.

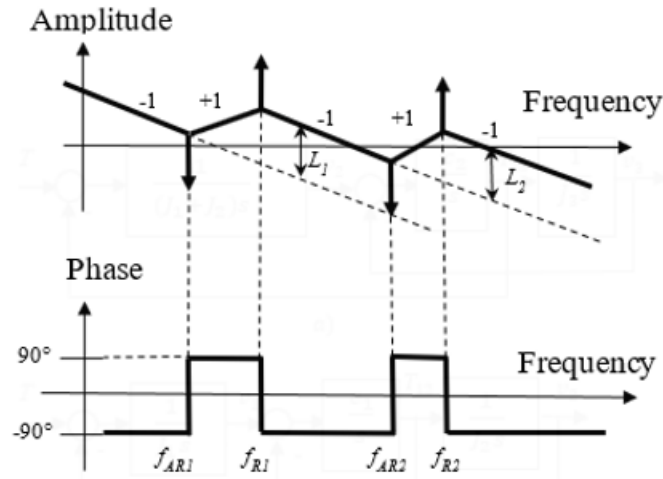


Figure 4: Asymptotic motor Bode plots of robot's arm.

By comparing P_{AR1} to P_{R1} and P_{AR2} to P_{R2} , we can find the following mismatches:

$$\left(\frac{P_{AR1}}{P_{R1}}\right)^2 = \frac{(J_1 + J_2 + J_3)}{(J_1 + J_2)}, \quad (11)$$

$$\left(\frac{P_{AR2}}{P_{R2}}\right)^2 = \frac{(J_1 + J_2)}{J_1}. \quad (12)$$

Denoting the mismatches for each body to first one as, we can $Y_2 = \frac{J_2}{J_1}; Y_3 = \frac{J_3}{J_1}$ re-write (11), (12) as:

$$\left(\frac{P_{AR1}}{P_{R1}}\right)^2 = (1 + Y_2 + Y_3) / (1 + Y_2); \left(\frac{P_{AR2}}{P_{R2}}\right)^2 = 1 + Y_2 \quad (13)$$

or

$$\left(\frac{f_{AR1}}{f_{R1}}\right)^2 = (1 + Y_2 + Y_3) / (1 + Y_2); \left(\frac{f_{AR2}}{f_{R2}}\right)^2 = 1 + Y_2 \quad (14)$$

The increments L_1, L_2 of Bode amplitude after each resonance are defined by mismatch ratios for each resonance as follows:

$$L_1 = 20 \lg \frac{1 + Y_2 + Y_3}{1 + Y_2}, L_2 = 20 \lg (1 + Y_2) \quad (15)$$

Thus, up to the frequency f_{AR1} the Bode slope is determined by the total moment of inertia $J = J_1 + J_2 + J_3$, in the range of frequencies $f_{R1} \dots f_{AR2}$ is determined by sum of moments of inertia $(J_1 + J_2)$, and after the frequency f_{AR2} is determined only by the 1st body (motor) moment of inertia J_1 . Therefore, it could be considered that after each resonance's frequency one of the bodies is switched off. I.e., the third body is switched off for frequencies higher than the first resonance frequency f_{AR1} . Due to that fact, it is not considered for the second resonance. Furthermore, the second body is switched off for frequencies higher than the second resonance frequency f_{AR2} .

Such phenomena with switching off of bodies after resonance leading to Gain increasing which is known as "reduction of effective inertia" [7, 11].

Identification Algorithm: Step-by-step

Based on the above, the algorithm of parameter's' identification for three bodies' model may be presented by the following sequence:

1. The total moment of inertia J [kg·m²] of the arm can be calculated with two ways from: transient response or experimental Bode.

- In case of transient response, the total moment of inertia J can be found as $J = T/a$, where T is torque at constant acceleration a .
- The total moment of inertia J from experimental Bode

$$J = \frac{K_A K_F K_E}{K_G},$$

where the total Plant Gain $20 \lg K_G$ can be found from the Gain Bode at angular frequency of 1 rad/s. The Gains of Plant components as current amplifier K_A [Apk/bit], motor torque constant K_T [N·m/Apk] and position resolution K_E [counts/rad] will be found from the Data Sheets of the motor and the motion controller.

2. First resonance mismatch:

$$\frac{1+Y_2+Y_3}{1+Y_2} = \left(\frac{f_{R1}}{f_{AR1}}\right)^2$$

3. Increment L_1 :

$$L_1 = 20 \lg \left(\frac{1+Y_2+Y_3}{1+Y_2} \right)$$

4. The sum of the first and second moments of inertia with known total moment of inertia J : $(J_1+J_2) = J \cdot \frac{1+Y_2}{1+Y_2+Y_3}$

5. Third body moment of inertia:

$$J_3 = J - (J_1 + J_2)$$

6. Coupling's stiffness of the third body:

$$C_2 = J_3 \cdot (2\pi \cdot f_{AR1})^2$$

7. The for the second resonance mismatch with previously calculated sum of the first and second moments of inertia:

$$1+Y_2 = \left(\frac{f_{R1}}{f_{AR1}}\right)^2$$

8. Increment L_2 :

$$L_2 = 20 \lg(1+Y_2)$$

9. The first body moment of inertia (motor's moment of inertia including stiffer bodies attached to the motor)

$$J_1 = (J_1 + J_2) \cdot \frac{1}{1+Y_2}$$

10. The second body moment of inertia:

$$J_2 = (J_1 + J_2) - J_1$$

11. The compliant coupling's stiffness between first and second bodies:

$$c_1 = J_2 \cdot (2\pi \cdot f_{AR1})^2$$

Case study: parameters calculation of robot arm model

The frequency responses of the arm's Plant (from current amplifier input to axis' motor speed) - see Figure

1 - were received with Frequency Analyzer build-in modern motion controller [12]. The arm motor speed was detected as first derivation of the motor encoder - rigidly attached to the motor shaft - position.

Initial Data:

- Total moment of inertia $J=25.9 \cdot 10^{-3} \text{ kg} \cdot \text{m}^2$
- Gearbox type: harmonic drive
- AR and R frequencies of the first resonance $R_1: f_{AR1}=2.9 \text{ Hz};$

$f_{R1}=5 \text{ Hz}.$

- AR and R frequencies of the second resonance: $R_2: f_{AR2}=380 \text{ Hz}; f_{R2}=590 \text{ Hz}$

Calculation of the robot arm model parameters:

1. The mismatch for the first resonance with (J_1+J_2) and J_3 bodies moments of inertia:

$$\frac{1+Y_2+Y_3}{1+Y_2} = \left(\frac{f_{R1}}{f_{AR1}}\right)^2 = 2.97$$

2. The amplitude increment L_1 :

$$L_1 = 20 \lg \left(\frac{1+Y_2+Y_3}{1+Y_2} \right) = 9.46 \text{ dB}$$

3. The sum of the first and second bodies moments of inertia (J_1+J_2)

$$(J_1 + J_2) = J \cdot \frac{1+Y_2}{1+Y_2+Y_3} = 8.72 \cdot 10^{-3} \text{ kg} \cdot \text{m}^2$$

4. The third body moments of inertia J_3

$$J_3 = J - (J_1 + J_2) = 17.18 \cdot 10^{-3} \text{ kg} \cdot \text{m}^2$$

5. The stiffness C_2 between third body coupled and stiff coupled (J_1+J_2) body moment of inertia:

$$C_2 = J_3 \cdot (2\pi \cdot f_{AR1})^2 = 5.7 \text{ N} \cdot \text{m} \cdot \text{rad}^{-1}$$

6. The mismatch for the second resonance with J_1 and J_2 bodies' moments of inertia

$$1+Y_2 = \left(\frac{f_{R2}}{f_{AR2}}\right)^2 = 2.49$$

7. The amplitude increment L_2 :

$$L_2 = 20 \lg(1+Y_2) = 7.93 \text{ dB}$$

8. The first body moments of inertia J_1

$$J_1 = (J_1 + J_2) \cdot \frac{1}{1+Y_2} = 3.5 \cdot 10^{-3} \text{ kg} \cdot \text{m}^2$$

9. Second body moments of inertia J_2 :

$$J_2 = (J_1 + J_2) - J_1 = 5.22 \cdot 10^{-3} \text{ kg} \cdot \text{m}^2$$

10. Stiffness C_1 between second and first bodies

$$c_1 = J_2 \cdot (2\pi \cdot f_{AR1})^2 = 29.8 \cdot 10^3 \text{ N} \cdot \text{m} \cdot \text{rad}^{-1}$$

Finally, we can point out that the compliant couplings leading to

the resonances of the robot arm are located between:

- The third body with moment of inertia $J_3=17.18\cdot10^{-3}\text{kg}\cdot\text{m}^2$ and stiff coupled bodies $(J_1+J_2)=8.72\cdot10^{-3}\text{kg}\cdot\text{m}^2$ are leading to the lower frequency resonance.
- The second body with moment of inertia $J_2=5.22\cdot10^{-3}\text{kg}\cdot\text{m}^2$ and motor with moment of inertia $J_1=3.5\cdot10^{-3}\text{kg}\cdot\text{m}^2$ are leading to the higher frequency resonance.

Therefore, we can sign out that the source of first lower resonance is the compliance of the robot arm with masses attached

on an arm end - for example robot "elbow" arm - and second higher one - the compliance of the harmonic drive gearbox itself.

Additional tips for multi-body model design

Using the same approach including the superposition principle and two-body compliant reducer model, a multi-body model can be designed as serially connected n -bodies [13]. These fourth and more bodies should be connected one to the other (Figure 5) using the same principle as the third body shown in Figure 2.

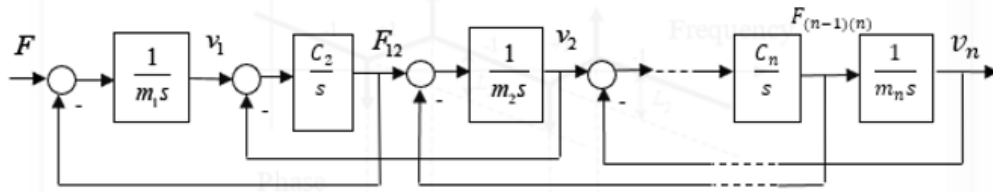


Figure 5: Block-diagram of a n -bodies model.

Identification of each resonance will be done by starting from the lowest frequency resonance to the higher ones. The mismatches of this n -body model could be calculated as:

$$\frac{1+Y_2+....Y_n}{1+Y_{n-1}} = \left(\frac{f_{R1}}{f_{AR1}}\right)^2; \frac{1+Y_2+....Y_{n-1}}{1+Y_{n-2}} = \left(\frac{f_{R2}}{f_{AR2}}\right)^2; \quad (16)$$

The successively switched off bodies after 1, 2,...,($n-1$) resonances have the according indexes $n,n-1,n-2,...,2$. Moments of inertia of the bodies with indexes $n,n-1,n-2,...,2,1$ and the coupling's stiffness $n-1,n-2,...,2,1$ may be found in the sequence using the suggested identification algorithm.

Conclusion

1. Modern motion controllers with built-in frequency analyzers allow for direct receipt of a frequency domain response (Bode) of a robot arm's plant with the real conditions [14].
2. The key features of the robot arm in the frequency domain are signed out for deep analyses of experimental Bode. And ones will be helpful for understanding the physic process cleaner.
3. The *2-bodies compliant reducer* model can be used to build a robot arm model in the case of 3- and more-bodies as well. Such model could help to understand a structure and identify parameters of most actual cases of n -bodies robot compliant arms.
4. The detection of key compliant couplings allows pinpointing the weakest ones, with a stiff point of view, to improve its

stiffness and thus, the performance of robot axes by increasing its servo bandwidth.

Acknowledgement

None.

Conflict of interest

No conflict of interest.

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