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Short Communication



# On the Compilation of Equations of Motion in Problems of Dynamics of Robots with Mecanum Wheels

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## Abstract

Kinematic conditions imposed on the movement of Mecanum robots with wheels lead to equations of nonholonomic constraints. For such constraints it is necessary to apply the equations of nonholonomic mechanics. Failure to take this circumstance into account may lead to errors. If we limit ourselves to the translational motion of the robot body and rotation around the center of mass, then the constraints become holonomic, the constraint equations can be solved for the velocities and then, Lagrange's equations of the second kind can be used.

**Keywords:** Mobile Robots; Mecanum Wheels; non-holonomic constraints

## Introduction

Since the development of the modern omnidirectional wheel by an engineer from the Swedish company Mecanum AB [1], such wheels have attracted the attention of researchers. A detailed presentation of the history of the appearance of Mecanum wheels is contained in [2]. This interest is due to the fact that robotic vehicles with Mecanum wheels have greater kinematic capabilities compared with robots with conventional wheels. They can rotate around the center of mass and move in any direction. Due to kinematic conditions, to describe the movement of robots with Mecanum wheels, it is necessary to use methods of nonholonomic mechanics, where dynamic equations in the general case differ from Lagrange's equations of the second kind.

## Discussion

The equations of kinematic constraints follow from the condition of the wheel rolling without slipping. For a normal wheel, this means that the velocity vector of the contact point must be zero. For a Mecanum wheel, only the projection of the contact point velocity vector $\boldsymbol{V}_P$ onto the roller axis vanishes [3]:

$$\boldsymbol{V}_P \cdot \boldsymbol{\gamma} = 0,$$

where $\boldsymbol{\gamma}$ is the unit vector along,the roller axis.

Such conditions lead to nonholonomic constraint equations when conditions are imposed on the generalized velocities. Let the position of the mechanical system be determined by $n$ generalized coordinates $q_i$, and let $n$-$m$ generalized velocities corresponding to these generalized coordinates be able to be expressed through the remaining $m$ generalized velocities using nonholonomic constraint equations:

$$\dot{q}_{m+k} = \sum_{s=1}^{m} b_{s,m+k}\dot{q}_s, \quad k=1,..,n-m,$$

where the coefficients $b_{s,m+k}$ can depend on all coordinates $q_i$. In the general case, these equations cannot be reduced to equations with respect to the generalized coordinates, that is, to geometric constraints (integrated). Thus, nonholonomic constraints impose restrictions on the generalized velocities without imposing restrictions on the generalized coordinates. In the general case, the answer to the question of when kinematic constraints can be reduced to geometric ones is given by the Frobenius theorem [4]. Generally speaking, Lagrange's equations of the second kind, the derivation of which uses holonomiciy of the constraints, are not suitable for describing the dynamics of mechanical systems with nonholonomic constraints. In some works this circumstance is ignored [5, 6]. To describe the dynamics of nonholonomic systems, the equations of Lagrange with multipliers, Voronets and others are used [7, 8]. If the coefficients $b_{s,m+k}$ in the equations of nonholonomic constraints depend only on the independent generalized coordinates $q_1,...,q_m$, then the Voronets equations are reduced to the Chaplygin's equations. A distinctive feature of Chaplygin's equations is the fact that they can be integrated independently of the constraint equations. The Voronets (Chaplygin) equations resemble Lagrange's equations of the second kind in appearance, but contain additional terms associated with the nonholonomicity of the constraints. In some rare cases, the sum of these additional terms may vanish even though each of them is not zero (in which case the constraints would be holonomic). Then the equations of motion under nonholonomic constraints coincide with Lagrange's equations of the second kind. This occurs, for example, for the equations of motion of a wheel pair [3]. The authors are not aware of any general theorems on this matter.

In some works, for example, [9, 10], the following technique is used: the equations of nonholonomic constraints are resolved with respect to the velocity vector using a pseudoinverse matrix. For a robot with four Mecanum wheels, there are four equations of nonholonomic constraints of the form:

$$\dot{\boldsymbol{\varphi}} = \boldsymbol{J} \cdot \boldsymbol{V},$$

where the vector $\dot{\boldsymbol{\varphi}}$ of the angular velocities of rotation of the wheels has a dimension of 4×1, the vector $\boldsymbol{V}$, consisting of projections of the velocity of the robot's center of mass onto the coordinate axes attached to the body, and the angular velocity of rotation of the robot's body has a dimension of 3×1, and the matrix $\boldsymbol{J}$ has a dimension of 4×3 . This system of equations is solved using a 3×4 pseudo-inverse matrix $\boldsymbol{J}^{+}$:

$$\boldsymbol{V} = \boldsymbol{J}^{+} \cdot \dot{\boldsymbol{\varphi}}, \quad \boldsymbol{J}^{+} = \left(\boldsymbol{J}^{T} \cdot \boldsymbol{J}\right)^{-1} \cdot \boldsymbol{J}^{T},$$

where $\boldsymbol{J}^{T}$ denotes the transposed matrix $\boldsymbol{J}$.

It should be noted that in the general case, the original system of equations as a system of four equations with three unknowns is inconsistent. Thus, the vector $\boldsymbol{V}$ found using the pseudoinverse matrix, generally speaking, does not satisfy the original system. In this case, the problem of solving the original system is replaced by the problem of minimizing the square of the Euclidean norm of the residual $\|\boldsymbol{J} \cdot \boldsymbol{V} - \dot{\boldsymbol{\varphi}}\|$. In [3] it is shown that for the translational motion of the robot and rotation around the center of mass, of the four equations of kinematic constraints, only three are independent, the original system has a solution, and the constraints become holonomic.

## Conclusion

The constraints imposed on a mechanical system with Mecanum wheels are in general non-holonomic due to the condition of rolling without slipping. To describe the dynamics of such systems, equations other than Lagrange's equations of the second kind are required. If we limit ourselves in advance to certain types of motions, such as translational motion of the body and rotation around the center of mass, then the constraints become holonomic and the Lagrange equations of the second kind are applicable. In the case of arbitrary trajectories of motion, it is necessary to use the equations of nonholonomic mechanics.

## Acknowledgements

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## Conflict of Interest

No Conflict of interest.

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