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Mini Review

Latest Research Trends of Set Pair Analysis in the Field of Water Environment

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Abstract

Set Pair Analysis (SPA) is a methodological framework for uncertainty analysis and has been widely used in water environment research. Integrating the latest concepts from quantum mechanics, this paper outlines recent research trends of SPA in the water environment and proposes a coupled Quantum-Set Pair Analysis (Q-SPA) evaluation method. Furthermore, it utilizes quantum entanglement mechanisms to optimize connection numbers and partial connection numbers, thereby providing a potential theoretical reference for uncertainty analysis and intelligent decision-making regarding complex water environment systems.

Key Words: Set Pair Analysis; Water Environment; Uncertainty; Quantum-Set Pair Analysis.

Introduction

Water environment systems are highly complex and uncertain. Traditional deterministic models cannot fully capture their uncertainty and dynamic evolution, highlighting the need for analytical approaches that can simultaneously address certainty and uncertainty. Set Pair Analysis (SPA) was proposed by Zhao Keqin in 1989 [1]. It uses the connection number, which combines identity, difference, and opposition, to quantitatively describe the certainty–uncertainty relationship within a system. The method has been widely applied in social sciences, engineering, and environmental sciences. Since its introduction into water environment research in 2000 [2]. SPA has evolved from its initial application in water quality assessment into a well-established theoretical and methodological framework with diverse environmental applications [3-4]. Building upon recent advances inspired by quantum mechanics, this paper reviews the latest developments in SPA for water environment research, with the aim of providing theoretical insights into uncertainty analysis in water environment systems.

Methodological Innovation: Fusion of Set Pair Analysis and Quantum State

In recent years, quantum theoretical concepts have received increasing attention [5]. The conceptual apparatus of quantum theory furnishes new analytical perspectives and mathematical formalisms for addressing uncertainty in system modeling. At the Partial Connection Number Intelligent Computing Workshop in November 2025, researchers focused on the quantum state and information transmission mechanism of partial connection numbers. Based on the commonalities of SPA and quantum mechanics in terms of discreteness and duality, superposition and collapse, a quantum-set pair analysis (Q-SPA) coupled evaluation model was constructed. Its fusion logic is as follows:

First, based on the Hilbert space representation of quantum superposition, three orthogonal quantum states, |G⟩, |M⟩, and |B⟩, are defined to represent three levels of "good, medium and bad".

Let the quantum state $|\Psi_{pq}\rangle$ corresponding to the -th indicator of sample be:

$$|\psi_{pq}\rangle = \alpha_{pq}|G\rangle + \beta_{pq}|M\rangle + \gamma_{pq}|B\rangle \quad (1)$$

where $|\alpha_{pq}|^2, |\beta_{pq}|^2, |\gamma_{pq}|^2$ correspond to the components of identity, difference, and opposition respectively, and satisfying

$$|\alpha_{pq}|^2 + |\beta_{pq}|^2 + |\gamma_{pq}|^2 = 1$$

Thus, the comprehensive quantum state of the water environment for sample p is:

$$|\Psi_p\rangle = \sum_{q=1}^{n} w_q |\Psi_{pq}\rangle = A_p|G\rangle + B_p|M\rangle + C_p|B\rangle \quad (2)$$

where w_q represents the weight of the q -th indicator among the indicators.

Furthermore, within the representation framework of quantum superposition states, a quantized partial connection state vector can be constructed to simultaneously characterize the superposition of multiple states and their evolutionary trends:

$$|\partial^+\mu\rangle = \partial^+ a|G\rangle + \partial^+ b|M\rangle + \partial^+ c|B\rangle,\ |\partial^-\mu\rangle = \partial^- a|G\rangle + \partial^- b|M\rangle + \partial^- c|B\rangle \quad (3)$$

$$|\partial^\pm\mu\rangle = |\partial^+\mu\rangle \oplus |\partial^-\mu\rangle = (\partial^+ a \oplus \partial^- a)|G\rangle + (\partial^+ b \oplus \partial^- b)|M\rangle + (\partial^+ c \oplus \partial^- c)|B\rangle \quad (4)$$

Specifically, by treating the interaction terms between the three states (e.g., ab and ba, bc and cb) as pairs of entanglement factors, a new formula for calculating partial connection numbers can be constructed. The specific calculation method requires further exploration [3].

Conclusion

SPA provides an effective framework for analyzing uncertainty and dynamic evolution in water environmental systems. This paper reviews recent advances in SPA inspired by quantum mechanics and proposes a coupled Q-SPA evaluation method. Future research will aim to clarify the physical interpretation of the proposed theoretical framework, enhance its environmental significance, and further optimize intelligent computing and co-evolutionary processes, thereby contributing to the smart regulation and resilient governance of watershed water environments.

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