



# The Non-linear Constitutive Modelling of Fibre Reinforced Composites

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## Abstract

The non-linear response of fibre reinforced composites has been long noticed. Numerous researchers have investigated into this subject and different non-linear constitutive model were proposed. This paper reviews some of the works done on the theoretical and numerical analysis of the non-linear response of FRPs. The developed models were examined, and deficiencies were discussed.

**Keywords:** Fibre reinforced composites (FRPs); Non-linear response; Constitutive model

## Introduction

The successful application of fibre reinforced composites (FRPs) in aerospace and automotive industries brings increasing requirements of understanding the failure behaviour of this material. However, the predictions of the mechanical response of composites were significantly affected by the non-linear behaviour of the material. This nonlinearity plays a critical part in accurate determination of the stress states before failure occurs. Therefore, thorough understanding of the non-linear stress-strain response under different stress conditions were essential.

## In-Plane Shear Nonlinear Modelling

Fibre reinforced composites (FRPs) usually exhibits linear elastic mechanical response along the fibre direction, while highly nonlinear stress-strain behaviour was reported under shear loading for carbon and glass fibre reinforced polymer composites [1], certain degree of nonlinearity was also detected on the transverse direction. Chang and Chang [2] simulated the progressive damage of open hole composites and suggested that the including of shear

nonlinear law plays an crucial part in accurately predict the tensile response of laminates.

To characterize the nonlinear behaviour of unidirectional (UD) laminates under in plane shear, Hahn and Tsai [3] proposed a one parameter polynomial shear stress-strain relationship:

$$\gamma_{ij} = \frac{\tau_{ij}}{G_{ij}^0} + \alpha \tau_{ij}^3$$

where,  $\gamma_{ij}$  and  $\tau_{ij}$  are the shear strain and stress, respectively,  $G_{ij}^0$  is the initial shear modulus,  $\alpha$  is the material parameter that can be determined from experimental results.

Several works have applied this model for shear damage analysis of composite materials. Chang and Lessard [4] used the Hahn-Tsai model to investigate the nonlinear response of open-hole laminates subjected to compressive loads. Dano et al. [5] utilized this model to taken into account the nonlinear shear response during the analysis of laminates with fastened joints. However, disadvantages were also

reported during the application of this nonlinear relationship: (1) the model was developed representing the nonlinear elastic shear behaviour, hence unsuitable for composite materials with large nonlinear shear deformation; (2) the form of the function uses stress to express strain makes it difficult to integrate into FE models.

Mohseni and Li [6] modified the three-rail shear fixture to perform loading-unloading cycle tests to study the nonlinear shear response of unidirectional composites. Experimental results suggest that the in-plane shear nonlinearity is composed of elastic, plastic and viscous shear deformation as well as shear hardening. Lou and Schapery [7] proposed a nonlinear viscoelastic constitutive relationship for unidirectional glass fibre composites based on the thermodynamic theories. The nonlinear viscoelastic parameters of this material were represented by the average octahedral shear stress of the matrix. Papanicolau et al. [8] applied Schapery's constitutive law to unidirectional composites under off-axis loading conditions. The nonlinear viscoelastic response of UD composite with three different fibre orientation angles ( $90^\circ$ ,  $75^\circ$  and  $60^\circ$ ) were investigated. Results indicate that as the decrease of fibre orientation angle, parameters of the constitutive equation increase.

Van Paepegem et al. [9] conducted monotonic and cyclic tension tests on  $\pm 45^\circ$  glass fibre reinforced polymers to characterise the in-plane shear behaviour of this material. The obtained nonlinear shear stress-strain relationship was identified as a permanent shear deformation and shear damage combination. A material model was built by Van Paepegem et al. [10] to simulate the permanent shear nonlinearity and shear modulus degradation of unidirectional and cross-ply composites. However, the effect of transverse stress on the evolution of shear damage was observed yet not analysed within this material model.

Totry et al. [11] conducted monotonic and unloading-reloading cycle tests on cross ply V-notch specimens to inspect the shear damage mechanisms. Shear deformation parallel and perpendicular to the loading direction was characterised, the overall shear performance was contributed by both elastic and plastic shear deformation. Totry et al. [12] further investigated the effect of properties of fibre, matrix and interface on the shear nonlinear response of composite materials through in-plane shear tests of cross-ply specimens as well as simulations of micromechanical models. Conclusions were drawn that the matrix yield strength and interface strength has dominated the shear nonlinearity.

Laux et al. [13] developed a plasticity model to describe the nonlinear behaviour of UD composites. The non-associative flow rule and Drucker-Prager yield function was integrated in the model to simulate the biaxial stress state responses. To characterise the material parameters in the model, a modified Arcan fixture was developed to produce tension/shear and compression/shear combined stress states.

McCarthy et al. [14] model the nonlinear shear behaviour by fit the experimental stress-strain curves using cubic spline

interpolation. High accuracy was reported when simulating the shear nonlinear response of composite materials, whereas the nonlinear transverse behaviour was observed in experiments yet not taken into consideration.

### **In-Plane Shear/Transverse Coupled Nonlinear Modelling**

To further understand the effect of nonlinearity in different directions, Sun and Chen [15] proposed an one parameter plastic material model, in where the transverse and in-plane shear stresses were used to express the quadratic yield function of unidirectional fibre reinforced composites. The stress-strain relationship was deduced from the effective stress and effective plastic strain relationship that fitted from off-axis tension tests. Yokozeki et al. [16] reported an existence of off-axis tension and compression asymmetry in carbon/epoxy composites. They modified the Sun and Chen model by adding a Drucker-Prager model [17] like parameter to characterise the different nonlinear behaviours in off-axis tension and compression. Experimental stress-strain relationships of off-axis compression specimens with different angles were used to fit into one effective stress-strain curve. However, the adopted associative flow rule and yield functions can bring non-physical transverse plastic strains when analysing the off-axis specimens under tension and compression.

Ladeveze and Le Dantec [18] suggested that the shear nonlinearity was generated from matrix plasticity and fibre/matrix interface debonding, while the transverse nonlinear response was supposed to be caused by matrix plasticity and microcracking. Hence, they further improved the Sun and Chen model [15] by couple the plasticity model with matrix microcracking and matrix-fibre debonding induced damage, the anelastic shear strains was characterised by effective stress and isotropic hardening was assumed. However, the model described requires extensive experiments to calibrate the materials parameters. Payan and Hochard [19] calibrated the Ladeveze and Le Dantec [18] model by static tension and tension/tension fatigue tests of  $\pm 45^\circ$  carbon fibre epoxy specimens. The model was simulated to verify the fatigue shear nonlinear behaviour of the cross-ply laminates. Donadon et al. [20] simplified the Ladeveze and Le Dantec [18] model, the new nonlinear shear damage model decomposes the shear strain into elastic and damage strain components. Similarly, the damage strain was separated into elastic damage strain and in-elastic damage strain. The shear stress was expressed by an experimentally fitted cubic polynomial of strain and provides accurate predictions of composites under shear loading conditions. Egan et al. [21] applied this shear model into finite element simulation of bearing damage of composite joints, good correlation with experimental results was reported. Chen et al. [22] also combined this shear nonlinear law with plasticity damage model to simulate the progressive damage of open hole composite plates. Although this model shows simplicity and efficiency, it fails to represent the matrix cracking and fibre/matrix interface debonding mechanisms. O'Higgins et

al. [23] investigated the influence of matrix plasticity and shear-transverse coupled plastic response on the Ladeveze and Le Dantec [18] model's predictions. Results indicate that the absence of plasticity has little effect on the simulation accuracy, while the shear-transverse coupled nonlinearity act as an important material characteristic.

Vogler et al. [24] and Hsu et al. [25] investigated the transverse compression and shear nonlinear responses of AS4/PEEK composites experimentally and numerically. An experimental study comprised of in-plane shear, transverse compression and shear/compression combined biaxial stress states were conducted. Inspection of the results show that both directions have significant nonlinear deformations, interaction between shear and transverse compression stress-strain nonlinearity was also reported. Micromechanical models of the fibre reinforced composite were established with an elastic-visco-plastic model implemented to represent all inelastic responses and damages induced nonlinearity of the matrix. Simulations of the unidirectional composite under pure shear, transverse compression and shear/compression combined biaxial loadings were performed. Although the predictions of simple shear and compression nonlinear responses correlate well with experimental results, the proposed model cannot accurately predict both shear and transverse compression nonlinearity under biaxial stresses. The complex interaction mechanisms between shear and transverse nonlinearity needs in-depth study.

## Conclusion

Most researches on the nonlinear shear modelling of fibre reinforced composites are focused on the two-dimensional stress state condition. Fewer work has been extended into three-dimensional modelling of the nonlinear response of composite materials. The proposed three-dimensional nonlinear models bring in material parameters that require sophisticated experiments to calibrate.

Despite the good correlation between published nonlinear constitutive models with experiments, the variety of experimental methods was limited. Most material models were developed for analysing certain loading conditions and were verified with only a few groups of tests. The scarcity of experimental validations requires diverse testing methods to intensely investigate the nonlinear deformation mechanisms especially under biaxial stress states and different hydrostatic pressures.

Although the in-plane shear nonlinearity was thoroughly investigated through experiments and numerical modelling, the interaction between shear/compression or shear/tension combined stress states still lacks study. Based on the current researches, no agreement has reached on whether transverse stress has an effect on the shear nonlinear behaviour. Hence, further research on the coupling behaviour of shear/transverse direction nonlinearity are critical in help understanding the mechanisms of nonlinearity of fibre reinforced composites.

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## Conflict of Interest

No conflict of interest.

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