



Research Article

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Mathematical Aspects of Special Theory of Relativity

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Abstract

To investigate why the maximum velocity in special theory of relativity is equal to the speed of light, a propagation of some signal with superluminal velocity is considered. Modified kinematics and Lorentz transformations of Maxwell's equations are described. It is shown that the only equality of maximum velocity in special theory of relativity to the speed of light conserves the invariance of Maxwell's equation in any inertial frames. That confirms the electromagnetic nature of the maximum speed in A. Einstein's special theory of relativity

Keywords:

Relativity; Lorentz transformations; Light speed; Maxwell equations

Introduction

Two great masterworks influenced on this study: A famous article "On the Electrodynamics of moving bodies" [1] by Albert Einstein and a perfectly written book "Ryman geometry and tensor analysis" [2] by Petr K. Rashevsky.

In this study we follow the conjecture, expressed by A. Einstein, "that not only in mechanics, but in electrodynamics as well, the phenomena do not have any properties corresponding to the concept of absolute rest, but that in all coordinate systems in which the mechanical equations are valid, also the same electrodynamic and optical laws are valid, as has already been shown for quantities of the first order" [1] (the first Einstein's postulate (A) or "principle of relativity"). The second Einstein's postulate of independence of the light speed in empty space of whether the ray of light is emitted by a body at rest or a body in motion is grounded on the experimental observations of A. A. Michelson and E. W. Morley [3–5]. To prove this not obvious postulate of the exceptional role of the light speed in nature, we attempt to construct the modified theory of special relativity with superluminal velocity of some signal and then will show why the maximum velocity in special theory of relativity should be equal to the speed of light. We modify the second Einstein's postulate in the way as follows: (B) there is an interaction between some objects in empty space that produces a "super" signal that is always propagated with a definite velocity S which exceeds the speed of light and is independent of state of motion of the emitted body (a principle of existence of the constant superluminal velocity of the interaction propagation).

Combining these principles (A) and (B) we modify special theory of relativity to allow the propagation of super signal (super light) with a velocity exceeding the light speed in empty space.

In Section 2 we obtain the modified Lorentz transformations of the space-time coordinates of the events when the transition from one inertial frame of reference to another one occurs. In Subsection 3.1 we analyse the modified Lorentz transformations and apply it to kinematics and dynamics. In Subsection 3.2 we transform the components of the electromagnetic field from the system at rest to a system of moving body. Finally, we draw the conclusions in Section 4.

We use SI units throughout all the study except of Subsection 3.2, where we use Gaussian System of equations.

Materials and Methods

Consider two Cartesian coordinate reference frames in a lab system in which the Newtonian mechanical equations are valid. Let the X -axes of these two frames coincide and their Y - and Z -axes be parallel. Let each system contains a rigid measuring rod and some clocks and let the two measuring rods and all the clocks of the two systems be exactly alike that means the units of length or the units of time in both systems we assume the same. Let the origins of two systems coincide at time equal to zero in both reference frames.

The origin of one of the two systems (K') shall now be imparted a constant velocity v in the direction of increasing x of the other system (K), which is at rest in the lab system, and we assume that in each time t the axes of the moving system are parallel to the axes of the system at rest. Thus, we investigate the case of motion of the reference frame K' relative to the laboratory frame K with a constant velocity $v_x = v$, $v_y = v_z = 0$.

Let to measure the space both from the system at rest K by means of the measuring rod at rest, and from the moving system K' by means of the measuring rod moving along with it. The coordinates x, y, z in the reference frame K and ξ, η, ζ in the reference frame K' corresponds to these measurements.

We now imagine that there are detectors A, B , and C mounted at equal distances d from the origin on the three axes of the moving system K' at the coordinate values $\xi = \eta = \zeta = d$ and these detectors can register a signal from some interaction occurred in the space point O where the origins of both systems K and K' coincided at the momentum of time $t = 0$. Let call this signal a ray of super light and let it propagate with a superluminal velocity S which is independent of state of motion of the emitted body, i. e. velocity of such a signal measured in the lab system K and in the moving system K' has the same numerical value.

We denote this velocity S like a distance d passed by the ray of super light divided by a time interval τ needed for that in the system at rest. In this way

$$S = \frac{\xi}{\tau} = \frac{\eta}{\tau} = \frac{\zeta}{\tau} = \frac{d}{\tau}, \quad (1)$$

because the detectors are at rest in the system K' and τ is a time interval needed for the ray of super light to be registered by the detectors in the system, where they are at rest.

In Figure 1 we draw the propagation of super light in the systems K and K' at different moments of time t , starting from $t = 0$, in a case when $v = 0.01 \cdot S$. For simplicity we display only two coordinate axes of both coordinate systems.

Consider the propagation of super light in the lab system K from an origin O to the detectors A and B which move together with the system K' with velocity v and sequentially change their positions from point A to points $A', A'',$ etc., and from a point B to points $B', B'',$ etc. The origin of system K' moves from the point O to points $O', O'',$ and so on in the same moments of time t . According to the postulates (A) and (B) the super light spreads in the laboratory frame K with the speed S in the form of a sphere centered at the initial point O , coincided at the initial moment of time $t = 0$ with the origin of coordinates O' of the moving frame K' , and in the moving frame the super light propagates with the same speed S in the form of a sphere centered at the moving origin of coordinates O' . The wave fronts of super light in the laboratory system K are shown in Figure 1 by two concentric quarters of circles corresponded to: 1) the position B of the detector B in the laboratory frame K at the momentum of time $t = 0$; and 2) the position A' of the detector A when the origine of the frame K' moved from position O to O' corresponding to the moment of time $t = \tau$. The propagation of super light in the laboratory frame K from origine O to the position B' of the detector B is shown in Figure 1 by a straight line.

Determine in system K a time interval t_x corresponded to a propagation of super light from its origin O to the detector A . When the ray of super light passes the distance $OA = d$ for a time interval $t_1 = \tau = d/S$, the detector A passes the distance $AA' = \tau v$, and to reach this position, the ray of super light needs additional time $t_2 = \tau v/S$. When the ray of super light passes the distance AA' for a time interval t_2 , the detector A passes the distance $A'A'' = \tau v^2/S$, and to reach new position of the detector, the ray of super light needs an additional time interval $t_3 = \tau v^2/S^2$ and so forth. Thus, we can express time t_x as a sequence of time intervals

$$\begin{aligned} t_x &= t_1 + t_2 + t_3 + t_4 + \dots = \tau \left(1 + \frac{v}{S} + \frac{v^2}{S^2} + \frac{v^3}{S^3} + \dots \right) = \\ \tau \left[1 + \frac{v}{S} + \left(1 + \frac{v}{S} \right) \frac{v^2}{S^2} + \dots \right] &= \tau \left(1 + \frac{v}{S} \right) \left(1 + \frac{v^2}{S^2} + \dots \right) = \\ &= \frac{\tau \left(1 + \frac{v}{S} \right)}{\left(1 - \frac{v^2}{S^2} \right)}. \end{aligned} \quad (2)$$

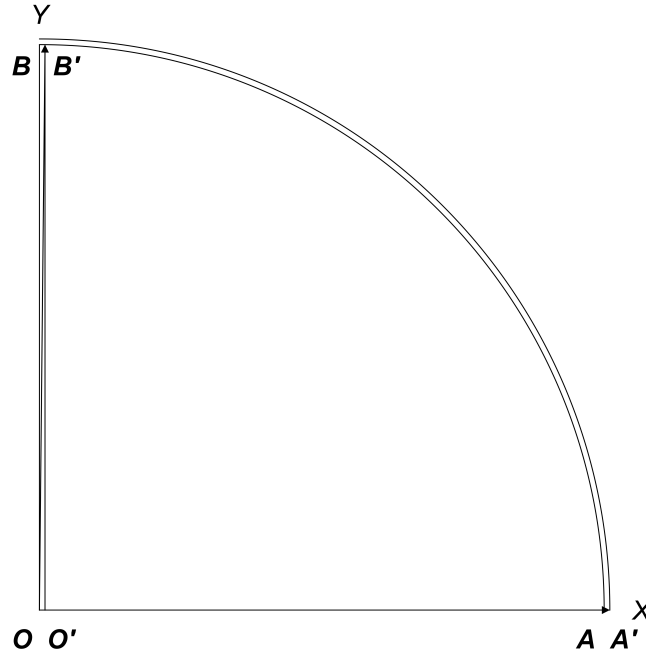


Figure 1: Propagation of super light with respect to the coordinate system at rest K when the system K' moves with a velocity $v = 0.01 \cdot S$. The position of the system K' corresponds to the time interval $t = \tau$. The wave fronts of super light in the laboratory system K are represented by two concentric quarters of circles corresponding to: 1) the position B of the detector B in the laboratory frame K at the moment of time $t = 0$; 2) the position A' of the detector A when the origin of the frame K' moved from position O to O' , corresponding to the moment of time $t = \tau$. The propagation of super light in the laboratory frame K from origin O to the position B' of the detector B is shown by a straight line.

If we use $\tau = \xi/S$ from Eq. (1) and assume $x = St_x$, then we obtain the expressions for t_x , and x as follows:

$$t_x = \frac{\left(\tau + \frac{v}{S^2}\xi\right)}{\left(1 - \frac{v^2}{S^2}\right)}. \quad (3)$$

$$x = \frac{(\xi + v\tau)}{\left(1 - \frac{v^2}{S^2}\right)}. \quad (4)$$

Determine time t_y that super light originated from O needs to be registered by detector B in the frame at rest K . When the ray of super light for a time t_y propagates with velocity S from origin O into direction of the detector B , it passes the distance $\sqrt{\eta^2 + v^2 t_y^2}$, that is equal to St_y . Thus, the ray of super light reaches detector B in a time interval

$$t_y = \frac{\eta}{\sqrt{S^2 - v^2}} = \frac{\tau}{\sqrt{1 - \frac{v^2}{S^2}}}, \quad (5)$$

if we apply $\tau = \eta/S$ from Eq. (1). A similar expression we obtain for a time interval t_z that super light needs to reach the detector C located at the distance d from the origin of the system K' on the Z' -axis:

$$t_z = \frac{\zeta}{\sqrt{S^2 - v^2}} = \frac{\tau}{\sqrt{1 - \frac{v^2}{S^2}}} = t_y. \quad (6)$$

Both time intervals t_y and t_z differ from t_x . From Eqs. (1), (2), (5), and (6) we obtain the ratios

$$t_x = \tau/(1 - v/S) = \xi/(S - v). \quad (7)$$

$$\frac{t_x}{t_y} = \frac{t_x}{t_z} = \frac{\sqrt{1 + v/S}}{\sqrt{1 - v/S}}. \quad (8)$$

From Eqs. (5) – (8) we derive that $t_x = t_y = t_z$ only in two cases: first, when $\xi = \eta = \zeta = 0$, and second, when $v = 0$. Thus, to keep the equality of those three time intervals we can apply two good approximations: first, $\tau \rightarrow 0$, and second, $S \rightarrow \infty$.

A. Einstein used in Ref. [1] the first approximation, considering an infinitesimal propagation of a signal assuming its

velocity to be equal to the velocity of propagation of electromagnetic interaction in empty space, $c = 3 \cdot 10^8$ m/s. In that case, if we assume the velocity v to be equal to the orbital velocity of the Earth $3 \cdot 10^4$ m/s, we obtain $t_x/t_y = t_x/t_z \approx 1.0001$, $t_x/\tau \approx 1.0001$, and $t_y/\tau = t_z/\tau \approx 1.000000005$.

In this study we use the second approximation $S \rightarrow \infty$. First, we refuse *the equality of the time and length units* in the systems K and K' and scale the time and length variables in the moving system K' with respect to system K in a way as follows:

$$t' = \gamma\tau, \quad x' = \gamma\xi, \quad y' = \gamma\eta, \quad z' = \gamma\zeta \quad (9)$$

where

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{S^2}}} \quad (10)$$

is the modified Lorentz factor. Since we use an approximation $S \rightarrow \infty$, the modified Lorentz factor $\gamma \rightarrow 1$, and the time and length units in the systems K and K' are almost the same.

These transformations conserve the numerical value of the velocity of super light in the moving system K' :

$$S = \frac{x'}{t'} = \frac{y'}{t'} = \frac{z'}{t'}. \quad (11)$$

Using scaling of Eq. (9) in Eqs. (3), (5), (6) we obtain the ratios

$$t_x = \gamma \left(t' + \frac{v}{S^2} x' \right), \quad t_y = t', \quad t_z = t'. \quad (12)$$

If we apply the principle (B) of the constancy of the velocity of super light S in any system at rest in particularly in system K , we obtain the ratios between the coordinates of two systems K and K' as follows:

$$x = St_x = \gamma \left(x' + vt' \right), \quad y = St_y = y', \quad z = St_z = z'. \quad (13)$$

To obtain the transformations of the coordinates and time when the transition from the moving system K' to the system at rest K occurs, we combine x' , y' , z' , and t' from Eqs. (12) and (13) in a quadratic form

$$s^2 = -S^2 t_x'^2 + x'^2 + y'^2 + z'^2. \quad (14)$$

This form is invariant in both systems K' and K :

$$s^2 = -S^2 t_x'^2 + x'^2 + y'^2 + z'^2 = \frac{(-S^2 t_x'^2 + x'^2) \left(1 - \frac{v^2}{S^2}\right)}{\left(1 - \frac{v^2}{S^2}\right)} + y'^2 + z'^2 = -S^2 t_x^2 + x^2 + y^2 + z^2 \quad (15)$$

If $s = 0$, then the coordinates x' , y' , and z' in the moving system K' lie on the sphere of radius St' centered at the origin of the coordinate system K' , and the coordinates x , y , and z in the lab system K lie on the sphere of radius St_x centered at the origin of the coordinate system K . The time interval t_x plays the same role in the lab system K as the time interval t' plays in the moving system K' : this is time of super light propagation from the origin of the reference frame to the detector A . Let call t_x a proper time in the system at rest K and denote it t .

Thus, when we obey the principles (A) and (B) formulated in Section 1, we obtain the modified Lorentz transformations of the space-time coordinates of events, when the transition from one inertial frame of reference to another one occurs:

$$t = \gamma \left(t' + \frac{v}{S^2} x' \right), \quad x = \gamma (x' + vt'), \quad y = y', \quad z = z'. \quad (16)$$

$$t' = \gamma \left(t - \frac{v}{S^2} x \right), \quad x' = \gamma (x - vt), \quad y' = y, \quad z' = z. \quad (17)$$

Here the primed coordinate x' , y' , z' and time t' are measured in the system K' moving with the velocity v with respect to the lab system K , and the unprimed coordinates x , y , z and time t are measured in the system at rest K .

In the modified Lorentz transformations (16) and (17) the *speed of light* c does not play an exceptional role anymore and becomes a velocity of propagation of one of the possible interactions. An exclusive velocity is the velocity of the propagation of super light S that has the same *numerical value* in all inertial frames of reference, in particularly, in the systems K and K' , and formally could exceed the velocity of the electromagnetic propagation. The propagation of the weak, strong, gravitational or any unknown interaction could be considered as the super light propagation.

Results

3.1. Modified Relativistic Kinematics and Dynamics

3.1.1. Geometrical Interpretation of the Modified Lorentz Transformations

In the system K' moving with velocity v along the X -axis of the system K , let there be a point moving with a velocity u' according to the equations

$$u'_x = \frac{dx'}{dt'}, \quad u'_y = \frac{dy'}{dt'}, \quad u'_z = \frac{dz'}{dt'}. \quad (18)$$

Differentiating the formulas (16) we obtain

$$dt = \gamma \left(dt' + \frac{v}{S^2} dx' \right), \quad dx = \gamma (v dt' + dx'), \quad dy = dy', \quad dz = dz'. \quad (19)$$

From these Eqs. (19) we can obtain the components of the point velocity on the axes X , Y , and Z of the system K as follows:

$$u_x = \frac{dx}{dt} = \frac{\left(v + \frac{dx'}{dt'} \right)}{\left(1 + \frac{v}{S^2} \frac{dx'}{dt'} \right)}, \quad u_y = \frac{dy}{dt} = \frac{\frac{dy'}{\gamma dt'}}{\left(1 + \frac{v}{S^2} \frac{dx'}{dt'} \right)}, \quad u_z = \frac{dz}{dt} = \frac{\frac{dz'}{\gamma dt'}}{\left(1 + \frac{v}{S^2} \frac{dx'}{dt'} \right)}. \quad (20)$$

Finally, we get

$$u_x = \frac{(v + u'_x)}{\left(1 + \frac{v}{S^2} u'_x \right)}, \quad u_y = \frac{\frac{u'_y}{\gamma}}{\left(1 + \frac{v}{S^2} u'_x \right)}, \quad u'_x, \quad u_z = \frac{\frac{u'_z}{\gamma}}{\left(1 + \frac{v}{S^2} u'_x \right)} \quad (21)$$

Consider a special case when u'_x is the velocity of propagation of the electromagnetic interaction in empty space, $c = 299792458$ m/s, and $v = 1.6 \cdot 10^4$ m/s. Let the ray of light is emitted in the direction of the movement of the system K' , then

$$c_x^+ = \frac{(v + c)}{\left(1 + \frac{vc}{S^2} \right)}, \quad c_y = c_z = 0. \quad (22)$$

If the ray of light is emitted in the direction opposite to the movement of the system K' , then

$$c_x^- = \frac{(v - c)}{\left(1 - \frac{vc}{S^2} \right)}, \quad c_y = c_z = 0. \quad (23)$$

The numerical value of c_x^+ is related to the numerical value of c_x^- by

$$\frac{|c_x^+|}{|c_x^-|} = \frac{c + v}{c - v} = 1.0001, \quad (24)$$

when $S \rightarrow \infty$. The ratio of the numerical values c_x^+ to c_x^- is equal to 1.00008 when $S = 2 \cdot c$

3.1.2. Geometrical Interpretation of the Modified Lorentz Transformations

From geometrical point of view equations (15) describe a 4-vector $\mathbf{x}$ in 4-dimensional pseudo Euclidian space of index 1 [2]. In orthonormal coordinate system x^0, x^1, x^2, x^3 , the scalar square of this 4-vector is

$$\mathbf{x}^2 = -x^{0^2} + x^{1^2} + x^{2^2} + x^{3^2}, \quad (25)$$

where

$$x^0 = St, \quad x^1 = x, \quad x^2 = y, \quad x^3 = z. \quad (26)$$

In this sense the modified Lorentz transformations (16) and (17) describe the rotation of the 4-vector $\mathbf{x}$ in 4-dimensional pseudo Euclidian space of index 1 that gives the transformation of coordinates x^0, x^1, x^2, x^3 to the coordinates

$$x^{0'} = St', \quad x^{1'} = x', \quad x^{2'} = y', \quad x^{3'} = z'. \quad (27)$$

The transition from one inertial frame of reference, K , to another one, K' , is equivalent to the transition from one orthonormal coordinate system in 4-space to another one. This transition could be described in a general case of the 4-vector $\mathbf{x}$ rotation and translation with a formula

$$x^{i'} = A_i^{i'} x^i + A^i, \quad (i = 0, 1, 2, 3), \quad (28)$$

where we assume a summation over twice appeared suffices. The pseudo orthonormal matrix $A_i^{i'}$ of order 4 index 1 is related with its inverse matrix $A_i^{i'}$ by [2]

$$\begin{pmatrix} A_0^0 & A_0^1 & A_0^2 & A_0^3 \\ A_1^0 & A_1^1 & A_1^2 & A_1^3 \\ A_2^0 & A_2^1 & A_2^2 & A_2^3 \\ A_3^0 & A_3^1 & A_3^2 & A_3^3 \end{pmatrix} = \begin{pmatrix} A_0^{0'} & -A_1^{0'} & -A_2^{0'} & -A_3^{0'} \\ -A_0^{1'} & A_1^{1'} & A_2^{1'} & A_3^{1'} \\ -A_0^{2'} & A_1^{2'} & A_2^{2'} & A_3^{2'} \\ -A_0^{3'} & A_1^{3'} & A_2^{3'} & A_3^{3'} \end{pmatrix} \quad (29)$$

In the case of the modified Lorentz transformations (17), that we rewrite in the form

$$x^{0'} = \gamma \left(x^0 - x^1 \frac{v}{S} \right), \quad x^{1'} = \gamma \left(-x^0 \frac{v}{S} + x^1 \right), \quad x^{2'} = x^2, \quad x^{3'} = x^3 \quad (30)$$

the inverse matrix A_i^j has a form

$$\begin{pmatrix} \gamma & \gamma \frac{v}{S} & 0 & 0 \\ \gamma \frac{v}{S} & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (31)$$

A curve in pseudo Euclidian 4-space, when depends on one parameter, e.g. $t = x^0/S$, describes the movement of particle in any inertial frame of reference [2]. If the velocity of the particle is less than velocity of super light S , then 4-trajectory of this particle lies inside an isotropic hyper cone and has an imaginary length. In each point of this 4-trajectory we can construct an imaginary unit tangential 4-vector

$$\tau = \frac{d\mathbf{x}}{d\sigma}, \quad \tau^2 = -1, \quad \sigma = \frac{s}{i}, \quad (32)$$

where $s = s(t)$ is imaginary length of the curve, i is imaginary unit, and σ is a real parameter.

The coordinates of 4-vector τ can be expressed in the form [2]

$$\tau^i = \frac{dx^i}{\sqrt{dx^{0^2} - dx^{1^2} - dx^{2^2} - dx^{3^2}}}, \quad (i = 0, 1, 2, 3). \quad (33)$$

3.1.3. Modified Relativistic Dynamics of Particle

Consider the particle moving in the inertial system at rest K with a velocity v . In this case we can tie to the particle a local inertial system K' moving with respect to the rest system K with the same velocity and apply all the formalism, developed above, to the transformation of variables, attached with the particle, when the transition from one system to another occurs.

If we assume the dependence of mass m of the particle from the velocity v in a form

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{S^2}}} \quad (34)$$

where m_0 is a mass of the particle at rest and S is a velocity of super light, then the total energy E of the particle is related to its mass m by

$$E = mS^2, \quad (35)$$

and the energy of the particle at rest E_0 is given by

$$E_0 = m_0 S^2. \quad (36)$$

In each point of 4-trajectory of the particle in pseudo Euclidian 4-space we construct a 4-momentum $E_0 \tau$ of the particle in a way as follows:

$$E_0 \tau^0 = mS^2, \quad E_0 \tau^1 = mv_x S, \quad E_0 \tau^2 = mv_y S, \quad E_0 \tau^3 = mv_z S. \quad (37)$$

where

$$v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}, \quad v_z = \frac{dz}{dt}. \quad (38)$$

The second Newton's law we can write then in the form

$$\mathbf{F} = \frac{d(m\mathbf{v})}{dt}, \quad (39)$$

i. e. a force $\mathbf{F}$ is equal to the derivative of momentum $m\mathbf{v}$ with respect to time t , where mass m is not a constant that gives an additional term in the differentiation.

3.2. Transformation of the Electromagnetic Field Components

Follow P. K. Rashevsky [2] consider a particle of charge e moving with a velocity v in the electromagnetic field at the rest system K . Let $\mathbf{E}$ (E_x, E_y, E_z) and $\mathbf{H}$ (H_x, H_y, H_z) be electric and magnetic field strength, respectively. In this field the particle experiences an action of Lorentz force

$$\mathbf{F} = e\mathbf{E} + \frac{e}{c} [\mathbf{v}\mathbf{H}] , \quad (40)$$

where c is an electromagnetic constant depended on the units of measurements. Quantitatively the constant c is equal to the speed of propagation of electromagnetic interaction in empty space measured at system at rest K .

According to the *principle of relativity* (A) we assume this force is valid in any inertial frame of reference. From (39) and (40) we derive an equation

$$\frac{d(m\mathbf{v})}{dt} = e \left\{ \mathbf{E} + \frac{1}{c} [\mathbf{v}\mathbf{H}] \right\} , \quad (41)$$

that we write in the form of the vector's components:

$$\begin{aligned} \frac{d(mv_x)}{dt} &= e \left\{ E_x + \frac{1}{c} (v_y H_z - v_z H_y) \right\} , \\ \frac{d(mv_y)}{dt} &= e \left\{ E_y + \frac{1}{c} (v_z H_x - v_x H_z) \right\} , \\ \frac{d(mv_z)}{dt} &= e \left\{ E_z + \frac{1}{c} (v_x H_y - v_y H_x) \right\} . \end{aligned} \quad (42)$$

If we multiply the terms of equations (42) by $S \cdot dt$ and consider equations (37), (38), and (26), then we obtain

$$\begin{aligned} d(E_0\tau^1) &= e \left\{ E_x dx^0 + \frac{S}{c} (H_z dx^2 - H_y dx^3) \right\} , \\ d(E_0\tau^2) &= e \left\{ E_y dx^0 + \frac{S}{c} (-H_z dx^1 + H_x dx^3) \right\} , \\ d(E_0\tau^3) &= e \left\{ E_z dx^0 + \frac{S}{c} (H_y dx^1 - H_x dx^2) \right\} . \end{aligned} \quad (43)$$

To these three formulas we add the fourth equation that expresses the differential of the particle energy mS^2 which is equal to the elementary work performed with the particle by the electric field strength:

$$d(E_0\tau^0) = e \{ E_x dx^1 + E_y dx^2 + E_z dx^3 \} . \quad (44)$$

Finally, we turn the coordinates of 4-vector $E^0\tau$ into covariant form and write equations (44) and (43) as follows:

$$d(E_0\tau_i) = eF_{ij}dx^j , \quad (45)$$

where antisymmetric tensor $F_{ij} = -F_{ji}$ has a form

$$\begin{aligned} \begin{vmatrix} F_{00} & F_{01} & F_{02} & F_{03} \\ F_{10} & F_{11} & F_{12} & F_{13} \\ F_{20} & F_{21} & F_{22} & F_{23} \\ F_{30} & F_{31} & F_{32} & F_{33} \end{vmatrix} &= \begin{vmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & \frac{S}{c}H_z & -\frac{S}{c}H_y \\ E_y & -\frac{S}{c}H_z & 0 & \frac{S}{c}H_x \\ E_z & \frac{S}{c}H_y & -\frac{S}{c}H_x & 0 \end{vmatrix} . \end{aligned} \quad (46)$$

This tensor permits to transform the components of the electric and magnetic fields from the lab system K to the moving system K' with a tensor transformation law as follows:

$$F_{i'j'} = A_{i'}^i A_{j'}^j F_{ij} , \quad (i, j = 0, 1, 2, 3) , \quad (47)$$

where the matrix $A_{i'}^i$ describes the transformation of the coordinates (28) and has the form (31).

Thus, the components of the electric and magnetic fields viewed from a frame moving with velocity v with respect to the lab system are given by

$$E'_{\parallel} = E_{\parallel}, \quad E'_{\perp} = \gamma \left(E_{\perp} - \frac{v}{c} H_{\perp} \right), \quad E'_z = \gamma \left(E_z + \frac{v}{c} H_y \right), \quad (48)$$

$$H'_x = H_x, \quad H'_y = \gamma \left(H_y + \frac{vc}{S^2} E_z \right), \quad H'_z = \gamma \left(H_z - \frac{vc}{S^2} E_y \right). \quad (49)$$

To make Maxwell's equations invariant in all the inertial frames of reference the velocity of super light S in transformations (48) and (49) should not differ much from the speed of light c in empty space. To estimate the value of the velocity of

super light we consider transformations (48) and (49) in a case when $v \ll S$. In this case $\gamma^2 \approx 1$ and we can rewrite the transformations (48) and (49) in the vector form

$$\mathbf{E}' = \mathbf{E} + \frac{1}{c} [\mathbf{v}\mathbf{H}], \quad \mathbf{H}' = \mathbf{H} - \frac{c}{S^2} [\mathbf{v}\mathbf{E}]. \quad (50)$$

There are two invariants [6], which are conserved when the electric and magnetic field components are transformed from the one inertial frame of reference to another:

$$\mathbf{E}' \mathbf{H}' = \mathbf{E} \mathbf{H}. \quad (51)$$

and

$$E'^2 - H'^2 = E^2 - H^2. \quad (52)$$

The transformations (48) and (49) conserve the form of Eqs. (51) exactly invariant. The transformations (50) also conserve the invariant (51) since term v^2/S^2 presented in the left part of Eq. (51) is negligible. The second invariant (52) is conserved when

$$\frac{1}{c} - \frac{c}{S^2} \rightarrow 0. \quad (53)$$

That means equality $S = c$ is really required to conserve the invariance of Maxwell's equation in any inertial frames.

Conclusions

The modification of special theory of relativity based on replacing in Lorentz transformations the speed of light by superluminal velocity changes the relativistic kinematics and breaks the invariance of two Maxwell's equations in the inertial frames. The only equality of the superluminal velocity to the speed of light conserves the invariance of Maxwell's equations in any inertial frames. This confirms the electromagnetic nature of the maximum speed in special theory of relativity.

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Conflicts-of-Interest

The authors declare no conflict of interest.

Use-of-AI-and-AI-Assisted-Technologies

No AI tools were utilized for this paper.

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