



Research on Decision-Making Problems in the Production Process of Enterprises —A Case Study of Problem B in the 2024 National College Student Mathematical Modeling Competition

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Received Date: March 19, 2026

Published Date: March 25, 2026

Abstract

In the production process of manufacturers, it is necessary to consider not only the production status of spare parts and the final completion degree of products, but also the key factor of product qualification rate. During manufacturing processes, manufacturers must consider not only component production conditions and final product quality but also incorporate the critical factor of product pass rate. By evaluating finished goods' pricing and quality based on consumer demand and market requirements, companies should analyse cost factors including inspection expenses, disassembly costs, and replacement expenses to determine whether to implement specific production operations. This study employs rational algorithms, utilizing hypothesis testing, confidence intervals, and rejection thresholds to derive optimal solutions. It identifies effective component inspection methods and pass rate testing strategies to maximize corporate profits. Additionally, the research provides actionable decision-making frameworks tailored to real-world scenarios, enabling businesses to optimize production and operational strategies through evidence-based approaches.

Keywords: Hypothesis Testing; Normal Distribution; Rejection Region; Confidence Interval

Introduction

With the legislation and implementation of the Extended Producer Responsibility (EPR) system, manufacturers are required to be responsible not only for the production process of products but also for the environmental responsibilities in the later stages of the product life cycle. The core of this system lies in shifting the environmental responsibility for products from traditional government management to the producers themselves, so as to incentivize manufacturers to design more environmentally friendly products and reduce resource waste and environmental pollution.

This requires enterprises to consider product quality and the remanufacturing issue after the end of product service life in the product design stage, so as to continue the performance and value of products.

Enterprise production decisions refer to the choices made by enterprises on what to produce, how much to produce, and how to produce when considering short-term production activities [9,13]. Their main task is to make the most reasonable and effective use of the enterprise's human, material, and financial resources

through the enterprise's own positioning under existing conditions to achieve the maximum economic benefits. Different production types have different effects on various enterprises, so enterprises must select the production type that best meets their requirements based on their actual situation.

Remanufacturing can not only extend the service life of products and increase their added value but also bring many benefits at the economic, environmental, and social levels. High-quality products can often attract more customers, increase market share, and thus boost sales [14]. Consumers are willing to pay higher prices for high-quality products, which can also improve the profit margin of enterprises. High product quality standards can also enhance brand image, strengthen consumer trust and loyalty, and thereby bring long-term economic benefits to enterprises. However, Fine [3] analysed that due to the learning curve effect, the higher the product quality, the greater the rate of production cost reduction, so that high-quality products obtain greater cost savings with the accumulation of product quantity compared to low-quality products. Gal-Or [4] analysed the relationship between product quality and output in a multi-firm market, and argued that under uniform demand distribution, it is necessary to reduce the average product quality to increase the total output, but none of these studies analysed the fixed cost investment in quality improvement. This paper mainly analyses the relevant data through hypothesis testing, rejection region, confidence interval [5-8] and other methods, and then puts forward suggestions for enterprises' production decisions.

Combining the above analysis, we take of the 2024 National College Students' Mathematical Modeling Competition as an example [11]. An enterprise needs to purchase two types of spare parts separately to produce electronic products, and assembles finished products by combining the two spare parts. If one of the spare parts in the finished product is unqualified, the finished product must be unqualified; however, even if both spare parts are qualified, the resulting finished product may not be qualified. For unqualified finished products, the enterprise can choose to scrap them or disassemble them at its own disassembly cost, but the disassembly process must ensure that the spare parts are not damaged. This study aims to derive the most suitable plan for the enterprise's development through hypothesis testing, confidence intervals, rejection regions, and other methods.

Model Assumptions

1. It is assumed that when inspecting semi-finished or finished products, the finished products are treated as a whole, and all semi-finished or finished products are inspected during the inspection, and vice versa.
2. It is assumed that the hardness, material, toughness and other properties of the same batch of purchased parts are basically the same.
3. No unexpected situations occur during the production operation process, ensuring production efficiency.

Model Establishment and Analysis

Hypothesis testing [5], which is generated to address the need for uncertain judgment of population parameters or distributions in practical research and decision-making processes, is an important method of statistical inference. It is applied in numerous fields such as scientific research, social surveys, economic

analysis, medical research, quality control, and decision-making. We often cannot directly obtain all the information about a population; instead, we can only infer the characteristics of the population from sample data. Hypothesis testing provides a scientific basis based on sample data for us, making decision-making more reliable.

In the process of an enterprise producing electronic parts and products, the goal is to design a sampling inspection plan with the minimum number of inspections as much as possible. Under the condition of meeting the nominal value of 10%, based on the designed sampling inspection plan, we respectively obtain the results under the condition of rejecting the parts if it is confirmed that the defective rate of parts exceeds the nominal value at a 95% confidence level, and accepting the parts if it is confirmed that the defective rate of parts does not exceed the nominal value at a 90% confidence level.

Model Establishment

Step 1: Propose hypotheses

The null hypothesis is denoted as H_0 . As the hypothesis that researchers aim to reject, the null hypothesis represents a status quo. The alternative hypothesis is denoted as H_1 , which represents the result the researcher expects to obtain.

- Under the 95% confidence level, if the measured defective rate of the spare parts exceeds the nominal value of 10%, we choose to reject this batch of parts.
- Under the 90% confidence level, if the measured defective rate of the spare parts is not higher than the nominal value, we choose to accept this batch of spare parts.

That is, when q satisfies the null hypothesis H_0 : the defective rate q of the parts \leq the nominal value $c=10\%$, we accept this batch of spare parts. When q satisfies the alternative hypothesis H_1 : the defective rate $c=10\%$, we reject this batch of spare parts.

Step 2: Select the test statistic

Choosing the appropriate test statistic in product decision-making is crucial. When selecting a test statistic, it is necessary to consider the population distribution, sample characteristics, sample size and other aspects to choose a suitable one, such as the common Z-test statistic and F-test statistic. Finally, according to the nature of the problem and the characteristics of the sample data, the Z-test statistic is selected, which is suitable for large sample sizes, known population variance and the Z-value distribution following the standardized normal distribution.

When the sample size is large, it can be approximately regarded as a normal distribution [12]. When the nominal value is known, the Z-test statistic is finally selected through screening.

Let the total sample defect rate be p , the number of samples drawn be n , the number of non-conforming items contained in them be X , and the nominal value, i.e., the defect rate critical value, be c . Then the sample defect rate can be obtained:

$$q = \frac{X}{n} \quad (1)$$

Since this sampling inspection is a random sampling, p follows a binomial distribution:

$$X \sim B(n, p) \quad (2)$$

When the number of sampled units is sufficiently large, it can be approximated as a normal distribution:

$$X \sim N(np, np(1-p)) \quad (3)$$

From equations (1), (2) and (3), its expectation and variance are respectively:

$$E(q) = \frac{E(X)}{n} = p; \quad \sigma(q) = \frac{\sigma(X)}{n^2} = \frac{p(1-p)}{n}$$

Then the normal distribution of the sample defective rate is derived:

$$q \sim N\left(p, \frac{p(1-p)}{n}\right).$$

After standardizing it, the final test statistic is obtained:

$$Z = \frac{q-p}{\sqrt{\frac{p(1-p)}{n}}} \sqrt{n} \sim N(0,1) \quad (4)$$

Step 3: Determine the significance level

Choose a significance level (α), such as 0.05 or 0.01 according to the specific situation of the problem. It represents the probability of the occurrence result when the null hypothesis is true. Under the 95% confidence level, ($\alpha = 0.05$); under the 90% confidence level, $\alpha = 0.1$.

Step 4: Calculate the sample size

In the first case, the batch of spare parts shall be rejected if $q > 0.1$ at the 95% confidence level. The rejection region is set

as $W = \{|u| > u_{0.95}\}$, and $u_{0.95} = 1.645$ under the standard normal distribution.

When the test statistic falls within the rejection region, i.e., $|Z| \geq u_{0.95}$. Let $q - c = \varepsilon$, which denotes the error within a controllable range, and take $\varepsilon = 0.05$. Then, at the 95% confidence level, the sample size is given by

$$n = \frac{u_{0.95}^2 c(1-c)}{\varepsilon^2}$$

Solving this yields $n = 97$.

In the second case, the batch of parts is accepted if $q \geq 0.1$ at the 90% confidence level. The rejection region is set as $W = \{|u| > u_{0.9}\}$, and $u_{0.9} = 1.282$ under the standard normal distribution.

When the test statistic does not fall within the rejection region, i.e., $|Z| \leq u_{0.9}$. Similarly, at the 90% confidence level, the sample size is

$$n = \frac{u_{0.9}^2 c(1-c)}{\varepsilon^2}$$

from which we obtain $n = 59$.

Step 5: Make a decision

At the 95% confidence level, a sample size of 97 is selected to determine whether to accept the batch of spare parts. If the defective rate exceeds the nominal value of 10%, the batch shall be rejected.

At the 90% confidence level, a sample size of 59 is selected to determine whether to accept the batch of spare parts. If the defective rate does not exceed the nominal value of 10%, the batch shall be accepted.

Decision Algorithm for Each Stage

The experimental data from Problem B of the 2024 Mathematical Modeling Contest are used for analysis. Based on the decisions made, specific decision schemes are provided for the cases in Table 1 below, together with the corresponding decision basis and indicator results.

During the inspection of spare parts, unqualified ones are directly discarded. In this case, the purchasing cost of qualified spare parts is increased. For example, when purchasing 10 units of Spare Part 1, the total purchasing cost is 40 yuan. If one of them is defective, only 9 qualified units of Spare Part 1 enter the assembly process. Thus, the purchasing cost per qualified Spare Part 1 is defined as $\frac{40}{9}$, approximately 4.444 yuan.

Table 1: Profit per finished product under each scheme.

Case	1	2	3	4	5	6	7	8
1	11.198	13.7755	11.22	14.3555	9.824	11.9156	9.36	11.9556
2	-2.2559	2.68	-1.82	5.1	-2.3279	1.84	-2.66	3.3
3	4.694	9.2156	6.66	11.9556	9.824	11.9156	9.36	11.9556
4	-13.968	-4.96	-8.46	3.3	-1.328	3.84	0.34	7.3
5	4.768	1.0756	9.96	7.4556	6.288	1.8756	9.86	6.4556
6	18.5867	19.3545	17.6176	18.5421	17.013	17.3295	15.5926	16.0421

Let x_1, x_2, x_3 denote whether Part 1, Part 2, and each assembled product are inspected, respectively; let x_4 denote whether defective finished products are disassembled. Let p_1, p_2, p_3 be the defective rates of Part 1, Part 2, and the finished product, respectively; let d_1, d_2, d_3 be their inspection costs, and W_2, W_3, W_5 the inspection expenses. Let d_4, d_5 be the unit purchase prices of Part 1 and Part 2; let W_4 be the total purchase cost of spare parts; let d_6 be the assembly cost of the finished product, d_7 the market selling price, and W_1 the sales revenue; let d_8 be the replacement loss for defective finished products, and W_6 the replacement cost; let d_9 be the disassembly cost; let p_0 be the probability that a finished product is successfully sold.

The expected unit purchase price of a qualified spare part is derived as follows: The probability that Part 1 is qualified in the first draw is $1 - p_1$; The probability that Part 1 is qualified only in the second draw is $p_1(1 - p_1)$.

The qualified probability when only the 1st spare part drawn for the third time is qualified is $p_1^2(1 - p_1)$;

The qualified probability when only the nth drawn part 1 is a qualified product is $p_1^{n-1}(1 - p_1)$.

Finally, the defined expected value of the cost of spare part 1 is

$$E = d_4(1 - p_1) + 2d_4p_1(1 - p_1) + \dots + nd_4p_1^{n-1}(1 - p_1).$$

We obtain

$$E = \frac{d_4(1 - p_1^n)}{1 - p_1} - nd_4p_1^n$$

The expected cost of part 1 defined as n approaches infinity is

$$E = \frac{d_4}{1 - p_1}.$$

Similarly, the expected value of the cost defined for spare part 2 is

$$E = \frac{d_5}{1 - p_2}.$$

One finished product is assembled using Part 1 and Part 2. An enterprise can only obtain profit when qualified finished products are sold in the market. Therefore, we first analyse the qualified rate of assembled finished products, that is, the probability of not being returned after entering the market. Let event A represent that Part 1 enters the assembly process, namely

$$P(A) = (1 - x_1)p_1 \tag{6}$$

Let event B represent that Part 2 enters the assembly process, namely

$$P(B) = (1 - x_2)p_2 \tag{7}$$

Thus, the unqualified rate of spare parts entering the assembly process is

$$P(A + B) = P(A) + P(B) - P(AB) \tag{8}$$

Even if both spare parts are qualified, the assembled finished product is not necessarily qualified. According to Table 1, the unqualified rate in this case is $p_3 = 10\%$, and the qualified rate is $1 - p_3$.

Then the probability of successful final sale is

$$p_0 = (1 - P(A + B))(1 - p_3) \tag{9}$$

From equations (6) (7) (8) (9), we can get

$$p_0 = (1 - (1 - x_1)p_1 - (1 - x_2)p_2 + (1 - x_1)(1 - x_2)p_1p_2)(1 - p_3)$$

The sales revenue obtained from each assembled finished product is

$$W_1 = d_7p_0.$$

The inspection cost for Part 1 is $W_2 = x_1d_1$; The inspection cost for Part 2 is $W_3 = x_2d_2$.

Finally, when choosing whether to inspect, the total purchase cost of spare parts is

$$W_4 = \frac{d_4}{1 - x_1p_1} + \frac{d_5}{1 - x_2p_2}$$

The inspection cost for the assembled finished product is $W_5 = x_3d_3$. The replacement loss for unqualified finished products returned by customers is

$$W_6 = (1 - x_3)(1 - p_0)d_8$$

To obtain a reasonable decision scheme, we need to maximize the profit, namely

$$\max : W = W_1 - W_2 - W_3 - W_4 - W_5 - W_6$$

The objective function is derived as:

$$W = d_7 p_0 - x_1 d_1 - x_2 d_2 - \frac{d_4}{1 - x_1 p_1} - \frac{d_5}{1 - x_2 p_2} - x_3 d_3 - (1 - x_3)(1 - p_0) d_8 \tag{10}$$

Given the defective rates of the two spare parts and the finished product, decisions are made for each stage of the enterprise's production process.:

- a) Whether to inspect the spare parts (Part 1 and/or Part 2). If a certain type of spare part is not inspected, all such parts directly enter the assembly process. If inspection is performed, any defective parts identified are discarded immediately.
- b) Whether to inspect each finished product. If no inspection is performed, all finished products directly enter the market, while unqualified products proceed to the next processing step.
- c) Whether to disassemble unqualified finished products. If disassembly is performed, the disassembled parts repeat

Steps a) and b) for further processing. If disassembly is not performed, unqualified finished products are discarded directly.

- d) For unqualified products returned by customers, the enterprise shall unconditionally replace them at its own cost, and the unqualified products shall repeat Step c).

The values of x_1, x_2, x_3 can be obtained by calculating Formula (10), which are used to determine whether to inspect the spare parts and finished products. When considering whether to disassemble unqualified products, a comparison is made between the sum of the disassembly cost and assembly cost $d_9 + d_6$ and the sum of the unit purchase prices of the spare parts $d_4 + d_5$. If $d_9 + d_6 > d_4 + d_5$, disassembly of unqualified finished products is rejected; otherwise, disassembly is adopted.

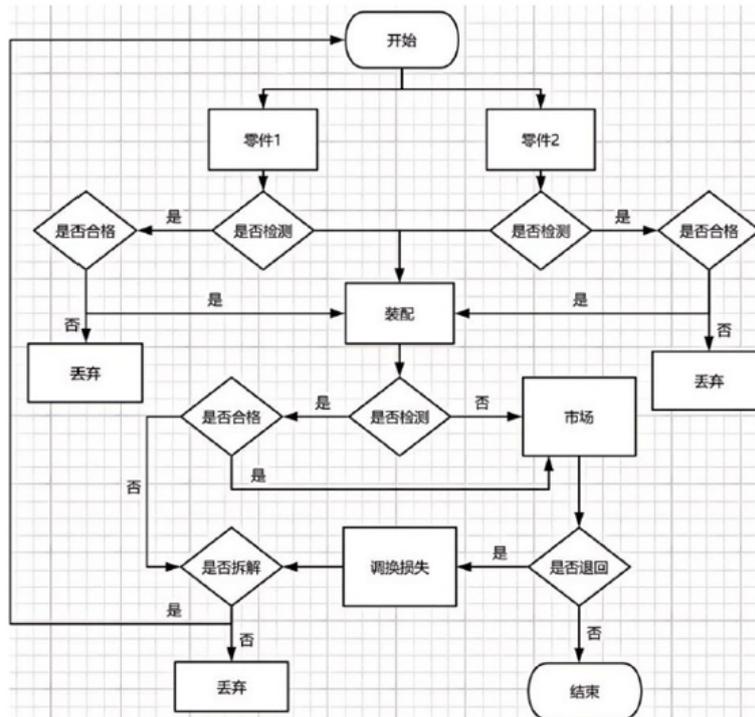


Figure 1: Flowchart.

By calculating the objective function, the corresponding values of x_1, x_2 and x_3 , under different scenarios are obtained. The

detailed results are presented in Table 2 below:

Table 2: Results of x_1, x_2, x_3 under different cases.

Case	Whether to inspect Part 1	Whether to inspect Part 2	Whether to inspect the assembled finished product
1	1	1	0
2	1	1	0
3	1	1	0
4	1	1	1
5	0	1	0
6	1	0	0

By comparing the magnitude of d_9 and $d_4 + d_5 + d_6$, the unqualified finished products in Cases 1 to 5 are disassembled, while those in Case 6 are not disassembled. Based on the above two decision criteria, the obtained decision schemes and corresponding indicators are as follows:

Case 1: Part 1 is inspected, Part 2 is inspected, the assembled finished product is not inspected, and the returned unqualified finished product is disassembled. The profit per finished product is 14.3556 yuan.

Case 2: Part 1 is inspected, Part 2 is inspected, the assembled finished product is not inspected, and the returned unqualified finished product is disassembled. The profit per finished product is 5.1000 yuan.

Case 3: Part 1 is inspected, Part 2 is inspected, the assembled finished product is not inspected, and the returned unqualified finished product is disassembled. The profit per finished product is 11.9556 yuan.

Case 4: Part 1 is inspected, Part 2 is inspected, the assembled finished product is inspected, and the returned unqualified finished product is disassembled. The profit per finished product is 7.3000 yuan.

Case 5: Part 1 is not inspected, Part 2 is inspected, the assembled finished product is not inspected, and the returned unqualified finished product is disassembled. The profit per finished product is 9.9600 yuan.

Case 6: Part 1 is inspected, Part 2 is not inspected, the assembled finished product is not inspected, and the returned unqualified finished product is not disassembled. The profit per finished product is 19.3545 yuan.

Conclusion

This paper uses hypothesis testing and probability statistics methods to make decisions at each stage of the production process. By conducting regular hypothesis tests, it assesses whether it is worthwhile to invest in more expensive raw materials or advanced production equipment, timely identifies problems in the production

process, helps enterprises conduct cost-benefit analysis, thereby improving product quality and

customer satisfaction. Furthermore, enterprises can evaluate the advantages and disadvantages of different solutions based on data and statistical analysis to make more scientific decisions. Compared with [10], we found that although the methods (tools) used were different, the resulting decision-making options were essentially similar.

Fund Project

National Natural Science Foundation of China (Nos. 11771007); Henan Natural Science Foundation (Nos.252300423520,262300421861) and Nanyang City Basic and Frontier Research Program Project.

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