Review Article

H. E. Kyburg Against the Heterodox Schools of Economics on the Issue of the Mathematical Lattice Structure Represented in Keynes's Diagram on p.39 of his Treatise (1921): it had nothing to do with Ordinal Probability

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Abstract

H.E. Kyburg correctly analyzed Keynes's page 39 illustration in a diagram presented in chapter III after having read pp.22-34 of Keynes's A Treatise on Probability. These pages involved discussions by Keynes of some applications of interval probability to conduct. Kyburg carefully analyzed Keynes's analysis on pp.34-40, showing that Keynes was describing a mathematical lattice structure consisting of interval probability. The Heterodox economists, on the other hand, did not read what Keynes had written, but, instead, decided to "interpret" Keynes through the foggy bifocals of F P Ramsey's confused and confusing 1922 and 1926 reviews, where Ramsey claimed that Keynes's "mysterious non numerical probabilities "were scientifically useless, ordinal probabilities. The following list provides a short list of the main heterodox economists advocating such an ordinal "interpretation". The rest of the members of all heterodox schools, who were working on Keynes's A Treatise on Probability, also all came to the same confused and confusing conclusion- that Keynes's theory of logical probability was an ordinal theory [1-11].

Kyburg basically published the same material on Keynes's diagram on page 39 of chapter III of the A Treatise on Probability four times, in 1995,1999,2003 and 2010, each time providing overwhelming analysis that the conclusion that Keynes's theory was based on ordinal probability made no sense technically. However, Kyburg's position made absolutely no headway among economists, philosophers or historians or other academicians. The reason it made no headway is the existence of very severe ,mathematical ,logical, probabilistic and statistical deficiencies that exist in all schools of heterodox economics involved in studying Keynes's A Treatise on Probability, A Treatise on Money ,the General Theory ,as well as all of his major post 1936 articles, such as the 1937 Eugenics Review and Quarterly Journal of Economics, the 1937-38 exchanges with H. Townshend, and the 1938-40 Economic Journal articles dealing with Tinbergen's application of the limiting frequency approach to probability to macroeconomic investment and business cycles.

Keywords: Interval probability; imprecise probability; upper-lower probability bounds or limits; non-numerical probability; mathematical lattice structure; ordinal probability; approximation; inexact measurement; JEL classification: D81

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Introduction

The paper will be organized in the following fashion. Section Two will cover Kyburg's analysis of how Keynes's diagram on page 39 (1921; p.42, 1973) of his A Treatise on Probability [12] shows that it is a mathematical lattice structure .This structure can be far more easily grasped by reading the problem at the end of Chapter XV on pp. 162-163 of Part II of the TP .The reader should also see some of the problems on pp.186-194 of the TP, where Keynes specifies additional least upper bounds and greatest lower bounds for Boolean problems. Section Three will cover the entire short list of heterodox economists ,who continue to promote the useless, intellectual confusion of F P Ramsey, by examining the latest advocate of the ordinal probability interpretation, (a) Zappia's 2025 book, which cites McCann's 1994 conclusion and (b) Carabelli's severe confusions which, in order to make any sense out of them, require one to read into Carabelli's pp. 24-32 [2] analysis Keynes's basic definition of a non-numerical probability, an example of which is an interval like [45, 57]. Kyburg's clear and straightforward summary of Keynes's argument on pp. 34-40, presented on pp. 400-401 of Kyburg (2003), makes it clear that an interpretation that Keynes is illustrating ordinal probability is simply wrong. Section Four examines the confused response to Kyburg made by Runde [13] and Zappia [5]. Section Five concludes the paper.

Kyburg's 2003 Analysis

Let me now present Kyburg's detailed, general, but much, much, longer analysis, which does not make use of the very easy result given on page 163 of the TP or pp. 186-194, to reach the conclusion that Keynes, following Boole, is specifying a mathematical lattice structure: "Although Keynes speaks repeatedly of "degrees" of rational belief, and acknowledges that to speak thus implies that degrees of belief are in some sense quantitative and "perhaps capable of measurement," he ends up by denying, in Chapter III, that all probabilities are even capable of comparison. In considering arguments in favor of the measurability of probabilities, Keynes (interestingly, vis a vis Ramsey) cites the of underwriters of insuring against almost any eventuality. "But this practice shows no more than that many probabilities are greater or less that some numerical measure, not that they themselves are numerically definite" [5, p. 22]. Keynes maintains"...that there are some pairs of probabilities between the members of which no comparison of magnitude is possible...[nor is it] always possible to say that the degree of our rational belief in one conclusion is either equal to, greater than, or less than our degree of belief in another"[5, p. 34]."... the closest analogy is that of similarity" [5, p. 36] This suggests, as does the illustration in [5, p. 39] and reproduced in slightly altered form in Fig. 1, that probabilities form a lattice structure [14-16].

a)   All probabilities are between 0 and 1.

b)   Some probabilities (Z and V, A and Y...) are not comparable.

c)   Probabilities fall on paths.

d)   One probability Z is less than another probability Y if it lies on a path between 0 and Y.

e)   A probability may belong to more than one path (W)".

Note that Kyburg's presentation of Keynes's diagram on p. 39 of the TP unfortunately has left out the non-numerical probability, W, from Keynes's diagram. Continuing on in his demonstration, Kyburg shows that the construction of interval valued probability follows directly from Keynes's own discussions made on pp. 34-40 of the TP.

"These probabilities do not satisfy the conditions for quantities; they are not magnitudes in the sense of lengths or masses. But they make sense! Here is model making use of two-vectors, though any multi-dimensional vector space would suffice.

a)   Construe probabilities as subintervals of [0,1].

b)   Take one probability interval to be less than another just in case every point in the first interval is less than any point in the second interval.

c)   When there is a point in one interval that is not in the other, and vice versa, we take the intervals to be incomparable.

d)   When every point of one interval is included in another interval, we take the intervals to be incomparable.

e)   Allow degenerate intervals.1

We now have a perfectly well-behaved lattice structure. The set of subintervals of [0,1] does form a lattice under the natural ordering $[p, q] \leqslant [r, s]$ if and only if every point in $[p, q]$ is less than any point in $[r, s]$. The meet of $[p, q]$ and $[r, s]$ is just the degenerate interval [min {r, p}, min {r, p}], and the join is the degenerate interval [max {s, q}, max {s, q}]. On this interpretation of the values of probability we have no difficulty in accommodating Keynes's graph. Furthermore, to the extent that probabilities are based on our knowledge of frequencies - and surely some probabilities are so based - it is natural to suppose that they are often valued intervals, since our knowledge of frequencies is inevitably approximate. In any event, it is clear that we can find a set of objects that has the structure that Keynes assigned to probabilities, and that this structure is consistent and coherent".

One can now see the impossibility of the diagram on p. 39 of the TP as having anything to do with being a representation of ordinal probability. The belief, that Keynes is talking about ordinal probability on p. 39 of the TP, as supposedly representing his theory of logical probability, is not founded on any type of analysis. It is simply an assertion made by economists. However, this very confused belief is the foundation for all economists' work on Keynes's TP, especially Post Keynesian and Heterodox economists.

In conclusion, there are two different ways to recognize that the diagram on page 39 is a mathematical lattice structure. The easiest way is to read chapter XV of the TP and connect it to Boole's analysis contained on pp. 287-325 of The Laws of Thought (LT, 1854) On the last two pages of chapter XV, pp. 162-163, of the TP, Keynes provides the results of his analysis. A reader can also verify Keynes's estimation of least upper bounds(lub's) and greatest lower bounds(glb's) by covering pp. 186-194 of chapter

XVII, which immediately leads to the conclusion that Keynes is talking about a mathematical lattice structure dealing with his sets of propositions involving the disjunctions and conjunctions of propositions. The other way is to follow Kyburg's detailed presentation, which is directly based on Keynes's discussions of "non numerical" probability on pp. 34-40 of the TP. I believe that reading pages 162-163 of the TP is the easiest and quickest way to discern that Keynes's diagram on p. 39 of the TP is a mathematical lattice structure.

Heterodox Confusions about Keynes's Diagram

Zappia's following footnote is very badly written. It creates additional confusions for a reader that are added to the confusions of McCann, which are based on the confusions of Ramsey. Zappia simply is unable to grasp that Keynes is talking about interval probability and not ordinal probability: "According to McCann [9] the figure makes it clear that, while the traditional view of probability is cardinal and one-dimensional. Keynes's view is ordinal and multidimensional. More technical, but still suggestive, the consideration made by Kyburg that Keynes's claim was that "the manifold of probabilities is richer than the set of real numbers between 0 and 1 [8].

Zappia fails to inform the reader of his chapter 3 that the positions of McCann and Kyburg are in direct conflict with each other, as McCann's claim is that Keynes's representation of probability is ordinal, while Kyburg proves that Keynes's representation was interval. Instead of Zappia's severe confusion about the nature of Kyburg's analysis, which proved, not only that Zappia and McCann, but also Skidelsky, Moggridge, Carabelli, O'Donnell, Runde, Davis, Fitzgibbons, Meeks, Basili and Zappia, etc., were wrong about the diagram on p.39 of the TP ,Zappia could have simply stated what Kyburg had actually accomplished ,which was that "...Keynes's claim was that ,as demonstrated by Kyburg, the diagram was a mathematical lattice structure composed of interval probability".

Carabelli [2] had covered the necessary individual pieces of Keynes's discussions on pp.30-34 that were required to allow a reader to follow Keynes in the construction of his illustration of a lattice structure. Carabelli understands the ordering of the series by the use of "between ", which requires the symbol "≤". She also understands that U, V, W, X, Y, and Z are non-numerical probabilities. She also understands that Keynes is using non numerical to mean non- measurable, non-comparable or incommensurable by a single, precise number. She understands that Keynes is making use here of his following written out analysis for readers who lack mathematical training and sophistication.

"This leads up to a contention, which I have heard supported, that, although not all measurements and not all comparisons of probability are within our power, yet we can say in the case of every argument whether it is more or less likely than not. Is our expectation of rain, when we start out for a walk, always more likely than not, or less likely than not, or as likely as not? I am prepared to argue that on some occasions none of these alternatives hold, and that it will be an arbitrary matter to decide for or against the umbrella"

[2] and "I maintain, then, in what follows, that there are some pairs of probabilities between the members of which no comparison of magnitude is possible; that we can say, nevertheless, of some pairs of relations of probability that the one is greater and the other less, although it is not possible to measure the difference between them; and that in a very special type of case, to be dealt with later, a meaning can be given to a numerical comparison of magnitude. I think that the results of observation, of which examples have been given earlier in this chapter, are consistent with this account. By saying that not all probabilities are measurable, I mean that it is not possible to say of every pair of conclusions, about which we have some knowledge, that the degree of our rational belief in one bears any numerical relation to the degree of our rational belief in the other; and by saying that not all probabilities are comparable in respect of more and less, I mean that it is not always possible to say that the degree of our rational belief in one conclusion is either equal to, greater than, or less than the degree of our belief in another".

Unfortunately, Carabelli does not understand that when Keynes is talking about non numerical, incomparable, unmeasurable or incommensurable probabilities, Keynes is talking about interval probabilities that overlap each other or where one interval is contained within the bounds of another interval. Examples are p1= [.43,.52] and p2= [.49,.55] or p1= [.40,.45] and p2= [.38,.52].

What Carabelli needed to be able to do was to convert these statements in Kyburg into Keynesian nonnumerical probabilities (Kyburg, 2003, pp.140-141): "These probabilities do not satisfy the conditions for quantities; they are not magnitudes in the sense of lengths or masses. But they make sense! Here is model making use of two-vectors, though any multi-dimensional vector space would suffice.

a) Construe probabilities as subintervals of [0,1].

b) Take one probability interval to be less than another just in case every point in the first interval is less than any point in the second interval.

c) When there is a point in one interval that is not in the other, and vice versa, we take the intervals to be incomparable.

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structure a la Boole in 1854 [17]. However, there is no economist, orthodox or heterodox, or philosopher, historian, etc., in the 20th or 21st centuries whoever read Part II or anything from chapters XV or XVII of the TP. The reason for this is one that I have been aware of since 1979-the acceptance of the many, various claims made by F P Ramsey, with no citations to Keynes's TP to back up his claims, about alleged errors made in Keynes's logical theory of probability .In fact ,there are no errors .The so called errors are confusions on the part of Ramsey, as they do not exist either in Keynes's A Treatise on Probability or in any other publication published by Keynes in his lifetime.

Economist Comments on Kyburg's Demonstration

There are two comments made by economists since Kyburg's four articles were published. The first comment is by Runde [13] on Kyburg's [14] version. The second comment is by Zappia [8] to Kyburg's 2003 version. Runde's comment is discussed below.

Runde states that "It is refreshing to see a contribution that not only takes Keynesian intuitions seriously but also takes some positive steps toward fleshing them out" but Keynes "...devotes most of the Treatise on Probability to developing a logic of qualitative comparisons of given RPI's (author's insert-RPI stands for relation of partial implication)". Of course, Runde has always been confused about Keynes's logical analysis of relational, propositional logic because he never read Boole's LT or understood Keynes's deployment of that logic in chapters I and II of the TP. There is a logic of qualitative comparisons in chapters IV, V and VI of the TP that is used to support Keynes's reformulated application of the Principle of Indifference (POI). However, nowhere else does Keynes make use of the POI as Keynes knew that it was very severely limited and constrained in application. Runde simply has no idea about what Kyburg's analysis is dealing with. However, Kyburg's refusal to make use of the high powered technical, mathematical analysis, available in the TP itself in Part II, meant that he could not completely refute Runde's Ramsey type conclusions.

The second comment is by Zappia: "According to McCann [9], the figure makes it clear that, while the traditional view of probability is cardinal and one-dimensional, Keynes's view is ordinal and multidimensional. More technical, but still suggestive, the consideration made by Kyburg, that Keynes's claim was that "the manifold of probabilities is richer than the set of real numbers between 0 and 1". Zappia's assessment of Kyburg, that "...More technical, but still suggestive, the consideration made by Kyburg that Keynes's claim was that "the manifold of probabilities is richer than the set of real numbers between 0 and 1" makes no sense at all. What Zappia has done is take a sentence out of context from Kyburg that would only make sense if a reader had already understood that Kyburg is talking about interval valued probability being represented by a mathematical lattice structure. If Zappia really wanted a reader to grasp what Kyburg was saying, then Zappia should have written that "...the consideration made by Kyburg that Keynes's claim was that interval probability is represented by a mathematical lattice structure. This is easy to understand for a reader who lacks basic mathematical training. Neither Runde

in 1995 or Zappia in 2025 appear to have grasped that Kyburg is simply translating Keynes's analysis on pp. 36-40 that results in the mathematical lattice structure presented by Keynes on page 39.

Conclusions

That the kind of major errors made by F P Ramsey in 1922,1923 and 1926 [18], regarding the work of Keynes (and also Russell), were never caught or challenged in the 1920's or 1930" s calls into question all claims made about the scientific and/or artistic nature of the social and behavioral sciences and liberal arts. Likewise, the blind acceptance of F P Ramsey's inchoate and confused claims by all academicians, including mathematicians, logicians and statisticians, working on Keynes's logical theory of probability for 104 years, also calls into question the scientific foundations of the entire body of "Keynes" scholarship. Examples of this literature are Bateman, Coates, Davis, Gerrard, Gillies, Janeway, Mellor, Mevthen, Misak and Runde [19-40]. The fact that the three contributions in 2025 on Keynes by M Coates, F Aristimuno and C Zappia, which are based on Ramsey's severe confusions, is evidence that the Social and Behavioral Science and Liberal Arts are NOT ABLE to correct the obvious errors made in Ramsey's work, or that of his followers, about Keynes (and Russell), for over one hundred years. Unfortunately, economists still lack the necessary mastery of mathematics and logic to understand what Keynes was doing in his diagram on p.39 of the TP. Kyburg's analysis should have been the end of the question about how to correctly interpret the diagram on page. 39. Unfortunately, economists still can't grasp Kyburg's analysis which incorporates the steps left out by Keynes in chapter III of the TP [41-105].

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