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# On Computational Modeling and Analysis of Transonic Aerodynamics

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## Abstract

There are many important research results in computational modeling and analysis of transonic aerodynamics over the years. The development of a mixed finite difference algorithm by Murman and Cole, for example, solves the steady transonic small-disturbance equation and captures the embedded shock waves naturally. Since then, many extensions and improvements have been made. These include the studies of different models involving higher dimension, moist air, and combustion. Algorithm using approximate factorization technique, in particular, allows accurate and efficient solution for the unsteady transonic case and is useful in realistic aircraft design.

**Keywords:** Transonic small-disturbance equations; Approximate factorization; Finite difference method

## Introduction

There has been considerable amount of researches conducted on the studies of transonic aerodynamics due to its use in airplane wing design. Using a general theory of expansion procedures, transonic flow past slender bodies and thin wings is investigated [1]. Asymptotic expansions with appropriate limiting procedures reduce the full equations to an approximation involving the steady transonic small-disturbance (TSD) equation. Expansion procedures of thin airfoil theory at various Mach numbers can be found in Section 5.3 of [2]. The TSD equation is mathematically interesting as the freestream Mach number approaching to unity it becomes a partial differential equation of mixed type (elliptic and hyperbolic). This creates a challenge for obtaining solution of the equation. There are several theoretical and computational methods based on the hodograph method. A series of interesting results for mixed subsonic-supersonic, shock-free flows are obtained in [3, 4]. Magnus and Yoshihara [5] computed mixed flows with shock waves by integrating the equations of unsteady compressible flow forward in time to approach a steady state. With a newly developed mixed finite difference method and a line relaxation algorithm, the transonic case of the equation is solved with shock waves that appear naturally [6]. Later, Murman [7] investigated the requirements for uniqueness of the calculated jump conditions across embedded shock waves and created the conservative Murman-Cole numerical method. The non-uniqueness of numerical solutions of potential equations at transonic speeds has been found to be related to the stability of the problem [8,9]. Kuzmin [10] reviewed all the solutions in detail where it was stated that all airfoils considered were long and flat with instability attributed to the rupture of supersonic regions.

Procedures have been developed for solving the TSD equations and full potential equations for two-dimensional and axisymmetric bodies and for the TSD equations for three-dimensional wings [11]. There are many notable extensions of the steady TSD equation model. We show just a few here. First, theoretical, and numerical studies of transonic flow of moist air around a thin airfoil are carried out [12-14]. In [14], steady TSD equation model with moist air and condensation on a thin airfoil is solved in order to investigate changes in the flow field by homogeneous nucleation of water and heat addition. Secondly, implicit methods, using successive line over-relaxation, alternating-direction and approximate factorization techniques, are adopted for solving the unsteady TSD equations [15-17]. Thirdly, a new small-disturbance model for a steady, lean, premixed combustion at transonic speeds in a channel of slightly varying area is presented in [18]. Finally, the TSD equations are coupled with aeroelastic solution in order to study the fluid-structure interactions [19, 20].

## Approximate Factorization Algorithm

To solve the unsteady TSD equations, explicit time-marching schemes were found to impose severe restrictions on the time step due to stability. Implicit schemes are thus adopted. The approximate factorization (AF) technique in [17], for example, is a time accurate algorithm formulated for solution of the three-dimensional unsteady TSD equation. The AF algorithm involves a time linearization procedure coupled with a Newton iteration technique. More specifically, for unsteady flow calculations, the solution procedure involves two steps. First, a time linearization step is performed to determine an estimate of the potential field. Second, Newton Iterations are performed to provide the time accuracy. To do that, the TSD equation is written in a general form as a nonlinear function of the unknown potentials at time level (n+1). The Newton iteration solution is then given by the first order Taylor series with the estimated potentials from the first step as the initial guess. Superior stability properties of the algorithm are demonstrated through applications to steady and oscillatory flows at subsonic and supersonic freestream conditions for an F-5 fighter wing. The AF algorithm is also shown to be efficient. It can provide accurate solutions in only several hundred-time steps, yielding a significant computational cost savings when compared to alternative methods. For reasons of practicality and affordability, an efficient algorithm and a fast computer code are requirements for realistic aircraft applications.

Several algorithm modifications have been made which have improved the stability of the AF algorithm and the accuracy of the results [21, 22]. A Computational Aeroelasticity Program - Transonic Small Disturbance (CAP-TSD) code permits the calculation of steady and unsteady flows about complete aircraft configurations for aeroelastic analysis in the flutter critical transonic speed range [19]. This CAP-TSD code uses the AF algorithm for solution of the unsteady TSD potential equation with five modifications: (1) an Engquist-Osher (E-0) [21] type-dependent switch to treat regions of supersonic flow, (2) extension of the E-0 switch for second-order spatial accuracy, (3) nonisentropic effects to treat strong-shock cases, (4) nonreflecting far field boundary conditions for unsteady applications, and (5) several modifications to accelerate convergence to steady state. Calculations are presented for several configurations including the General Dynamics one-ninth scale F-16C aircraft model to evaluate the modified algorithm. These modifications have been shown to significantly improved the stability of the AF algorithm and hence the reliability of the CAP-TSD code in general. Calculations are also presented from a flutter analysis of a 45” sweptback wing which agree well with the experimental data. The results and comparisons demonstrate the stability, accuracy, efficiency, and utility of CAP-TSD.

## Conclusion

For steady TSD equations, mixed finite difference method with a line relaxation captures the embedded shock waves for the transonic case. For unsteady TSD equations, AF algorithm coupled with computational aeroelasticity program provided accurate and efficient solution for engineering design applications. Comparisons to experimental data serves as a check for the computational methods and it also provides clues for algorithm invention and modifications.

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## Conflict of Interest

No conflict of interest.

## References

1. Cole JD, Messiter AF (1957) Expansion Procedures and Similarity Laws for Transonic Flow. *Zeitschrift fur angewandte Mathematik und Physik* 8(1): 1-25.
2. Kevorkian J, Cole JD (1981) Perturbation Methods in Applied Mathematics. *Applied Mathematical Sciences, Volume 34, Editors: F. John, J. P. LaSalle, and L. Sirovich,* Springer-Verlag New York.
3. Nieuwland GY (1967) Transonic potential flow around a family of quasi-elliptical aerofoil sections. *NLR Technical Report T* 172.
4. Korn DG (1969) Computation of shock-free transonic flows for airfoil design. NYO-1480-125, *New York University, Courant Institute of Mathematical Sciences.*
5. Magnus R, Yoshihara H (1970) Inviscid transonic flow over airfoils. *AIAA Journal 8*(12): 2157-2162.
6. Murman EM, Cole JD (1971) Calculation of plane steady transonic flows. *AIAA journal 9*(1): 114-121.
7. Murman EM (1974) Analysis of embedded shock waves calculated by relaxation methods. *AIAA journal 12*(5): 626-633.
8. Williams MH, Bland SR, Edwards JW (1985) Flow instabilities in transonic small-disturbance theory. *AIAA journal 23*(10): 1491-1496.
9. Liu Y, Luo S, Liu F (2017) Multiple solutions and stability of the steady transonic small-disturbance equation. *Theoretical and Applied Mechanics Letters 7*(5): 292-300.
10. Kuzmin A (2012) Non-unique transonic flows over airfoils. *Computers & Fluids* 63: 1-8.
11. Yoshihara, H (1973) A survey of computational methods for 2D and 3D transonic flows with shocks. GDCA-ERR-1726, Convair Aerospace Division, General Dynamics.
12. Campbell JF, Chambers JR, Rumsey CL (1989) Observation of airplane flow fields by natural condensation. *J. Aircraft* 26, pp. 593–604.
13. Schnerr GH, Dohrmann U (1994) Drag and lift in non-adiabatic transonic flow. *AIAA J* 32: 101-107.

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14. Lee JC, Rusak Z (2002) Theoretical and numerical studies of transonic flow of moist air around a thin airfoil. *Theoretical and Computational Fluid Dynamics 15*(6): 359-372.
15. Axelsson O (2004) Encyclopedia of Computational Mechanics. Vol. 1, *Fundamentals*. John Wiley & sons, ltd.
16. Borland CJ, Rizzetta DP (1982) Nonlinear transonic flutter analysis. *AIAA Journal* 20(11): 1606-1615.
17. Batina JT (1988) Efficient algorithm for solution of the unsteady transonic small-disturbance equation. *Journal of Aircraft 25*(7): 598-605.
18. Rusak Z, Lee JC, Choi JJ (2008) A small-disturbance model of transonic combustion. *Combustion Theory and Modelling 12*(1): 93-113.
19. Batina JT, Seidel DA, Bennett RM, Cunningham HJ, Bland SR (1989) Steady and unsteady transonic small disturbance analysis of realistic aircraft configurations. Transonic Symposium: Theory, Application, and Experiment 1(2): 467-496.
20. Van Opstal TM, Hulshoff SJ, Verhoosel CV (2011) A robust solution procedure for the transonic small-disturbance equation in fluid-structure interactions. *The Open Aerospace Engineering Journal* 4: 1-10.
21. Engquist BE, Osher SJ (1980) Stable and entropy satisfying approximations for transonic flow calculations. Mathematics of Computation 34(149): 45-75.
22. Batina JT (1988) Unsteady transonic small-disturbance theory including entropy and vorticity effects. AlAA Paper No. 88-2278**,** Presented at the AIAA/ASME/ASCE/AHS/ASC 29th Structures, Structural Dynamics, and Materials Conference, Williamsburg, Virginia.

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