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Review Article

Influence of Beam-to-Column Stiffness Ratio and Semi-Rigid Joint stiffness on the Buckling Resistance of Steel Frames

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Abstract

This study explores the effect of the beam-to-column stiffness ratio and the rotational stiffness of semi-rigid beam-to-column joints on the buckling resistance of steel frame structures. The focus is placed on steel portal frames, with the aim of assessing how the interplay between structural geometry and connection flexibility influences the overall stability response of the system. A parametric numerical investigation is conducted by systematically varying the relative stiffness of beams and columns as well as the degree of semi-rigidity at the joints. For each configuration, linear buckling analyses are performed in order to determine the corresponding critical buckling loads. These results are then evaluated to identify the key parameters governing the stability performance of the frame. The findings show that both the beam-to-column stiffness ratio and the rotational stiffness of the connections have a substantial impact on the buckling capacity of the structure. In general, higher joint flexibility leads to a reduction in the critical load, while more favourable stiffness distributions between members can improve the global structural response. Overall, the study provides deeper insight into the stability and post-buckling behaviour of steel portal frames with semi-rigid connections and offers valuable guidance for their efficient design and optimization.

Keywords: Buckling; critical buckling load; columns; steel portal sway frames; stiffness coefficients; semi-rigid connection

Introduction

The main objective of the present study is to examine the buckling response and overall stability behavior of steel sway portal frames through a detailed parametric investigation. Special emphasis is placed on evaluating how important structural parameters influence the critical buckling load of the system. More specifically, the effects of the beam-to-column stiffness ratio, the roof inclination angle, and the rotational stiffness of semi-rigid beam-to-column connections are investigated. The study seeks to provide a better understanding of the interaction between structural geometry, member stiffness, and connection flexibility, as well as their role in the global stability performance of steel portal frames.

The phenomenon of buckling and the instability behavior of steel structures have attracted significant research interest over the years because of their major influence on structural safety and design efficiency. Stability analysis is considered a fundamental aspect of structural mechanics and steel design, since instability effects may considerably reduce the strength and load-carrying capacity of structural members and systems. Numerous studies have focused on the behavior of steel elements and frame structures subjected to compressive forces, employing theoretical approaches, experimental testing, and numerical analysis methods. In recent years, the advancement of computational techniques has enabled more accurate prediction of instability phenomena, taking into account the effects of geometric imperfections, material and geometric nonlinearities, and the flexibility of structural connections.

The evaluation of the buckling resistance of a column subjected to axial compression can be performed using either analytical or numerical approaches. The most commonly used analytical procedures are the effective length method and the notional load method [1], which have long been adopted in structural engineering practice (Connor [2], Chwalla [3], Petersen [4], [5], Pflüger [6], Rubin [7]) and are still incorporated in modern design standards such as Eurocode 3 [8, 9] and LRFD [10]. These methods, developed to facilitate hand calculations, rely on several simplifying assumptions that may significantly affect their accuracy.

In recent years, the rapid advancement of computer hardware and engineering software has enabled the application of more refined numerical methods. These approaches allow the determination of buckling resistance through both linear and nonlinear analyses, accounting for geometric nonlinearity (large displacements) and/or material yielding (Chen and White [11], White and Hajjar [12], [13], [14], Torkamani et al. [15], White and Hajjar [16], Chan [17], Torkamani and Sonmez [18], Kim et al. [19], Rubin [20]). Despite these developments, most practicing structural engineers continue to rely on analytical methods, particularly during preliminary design stages.

The theoretical foundation of Eurocode 3 was established based on the work of Wood [21]. Cheong-Siat-Moy [22] investigated the k-factor paradox in leaning columns, highlighting that buckling strength depends not only on the rotational boundary conditions of the member but also on the overall structural behavior. Bridge and Fraser [23] introduced an iterative method for determining effective length, incorporating axial forces in restraining members and allowing for negative rotational stiffness values. Cheong-Siat-Moy [24] further proposed a fictitious lateral load approach for assessing the strength of leaning columns. Analytical expressions for effective length determination in sway, non-sway, and partially sway frames, including leaning columns, were developed by Aristizabal-Ochoa [25], [26].

Hellesland and Bjorhovde [27] introduced a restraint demand factor that accounts for vertical and horizontal interaction effects in member stability, and later proposed an effective length method suitable for cases with significant stiffness variations, such as top or bottom stories in braced frames [28]. Kishi et al. [29] derived analytical expressions for effective length evaluation of columns with semi-rigid joints in sway frames, while Essa [30] developed a design approach considering varying story drift angles in unbraced multistory frames. The buckling response of metallic members is strongly influenced by the rotational stiffness of adjacent members at the upper and lower joints. Consequently, the contribution of each member's rotational restraint, depending on its end conditions, can be derived using the slope-deflection method [31]. Cheong-Siat-Moy [32, 33, 34] further contributed to the conceptual understanding of leaning column behavior at buckling initiation, extending the analysis to general cases with lateral restraint varying over a wide range. Karamanos and Zissopoulou [35] compared Eurocode 3 and LRFD provisions for sway frames, while Gantes and Mageirou [36], and Mageirou and Gantes [37, 38, 39] developed improved stiffness distribution factors and analytical expressions for effective buckling length estimation in multi-story frames with varying sway characteristics and semi-rigid connections. Finally, Kounadis et al. [40] examined the buckling strength of members using nonlinear analyses incorporating large displacements and material nonlinearity, supported by both analytical and numerical approaches.

Christopher and Bjorhovde [41] carried out analyses on a set of semi-rigid frames having identical geometry, loading conditions, and member dimensions, but differing in connection properties. Their study demonstrated how connection characteristics influence internal member forces, global frame stability, and interstorey drift. Jaspart and Maquoi [42] discussed the implementation of both elastic and plastic design approaches for braced frames incorporating semi-rigid connections. The buckling failure of steel reticulated domes with semi-rigid joints was examined by Kato et al. [43], based on a nonlinear elastic–plastic hinge model for three-dimensional beam-column elements, in which elastic–perfectly plastic hinges were located at the member ends and mid-span.

Lau et al. [44] conducted an analytical investigation on subassemblages with various semi-rigid connections subjected to different loading conditions and test configurations. Their results indicated that significant changes in the moment–rotation (M–φ) relationship have only a minor effect on the load-carrying capacity of both the column and the overall subassemblage. In a related study, Lau et al. [45] proposed a column design method for non-sway bare steel frames that explicitly incorporates the semi-rigid behavior of beam-to-column connections. Li [46] derived closed-form solutions of the second-order differential equation governing non-uniform members with both rotational and translational spring supports, covering eleven key cases. Based on extensive numerical analyses, Li and Mativo [47] proposed a simplified approach for estimating the ultimate load of semi-rigid frames in the form of multiple linear regression equations relating maximum load to geometric and section parameters. Liew et al. [48] developed a comprehensive database of moment–rotation relationships in terms of stiffness and moment resistance, enabling comparative evaluation of frame performance under different connection types. Reyes-Salazar and Haldar [49] performed nonlinear time-history seismic analyses on steel frames with semi-rigid connections subjected to thirteen earthquake records. They introduced a parameter, the T ratio, to quantify connection rigidity, defined as the ratio between the moment demand from beam-line theory and the fixed-end moment of the girder. Finally, Rodrigues et al. [50] investigated the equilibrium paths of both braced and unbraced steel plane frames with semi-rigid connections using a hybrid numerical algorithm that combines the convergence advantages of the iterativeincremental tangent method with the evaluation of unbalanced forces accounting for rigid body motion of the elements.

Steel Structure under Consideration

Initially, the analysis and design of a steel portal frame industrial shed with plan dimensions of 10 m × 55 m is carried out using SAP2000 software [51]. The steel frames are spaced at 5 m intervals and consist of columns with a height of 5 m (h = 5 m) and a bay width of 10 m (L = 10 m). The steel grade used is S235. Each frame is assumed to be sway, with fixed supports at both columns' ends. The frame is designed to satisfy the provisions of Eurocodes 0 and 3 [8, 9] for the typical loading conditions specified in Eurocodes 1 [52, 53] and 8 [54]. More specifically, the nominal loads acting on the frame are as follows (Figure 1, Table 1).

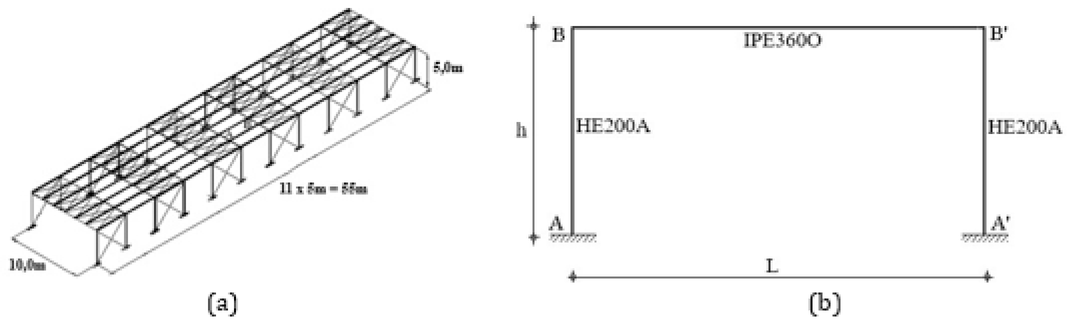

Figure 1: a) Steel structural industrial building, b) Steel portal frame.

Table 1: Parameters for the design of steel frame.

Panels load	0,12 kN/m^2
Live load (non-accessible roof)	5 kN/m
Snow load	1,6 kN/m
Wind load: wind pressure on the windward column	2,5 kN/m
Wind load: wind pressure suction on the leeward column	1,5 kN/m
Wind load: wind pressure suction on the rafter	1,0 kN/m
Temperature variation ΔT	±20°C
Ground acceleration	0,24 g
Response spectrum	EC8
Ductility class	low
Behaviour factor	1,15
Capacity design	yes
Connections classification	continuous

After completing the strength and serviceability checks in accordance with the provisions of Eurocode 3 [19], the final selection of the structural members was carried out. The adopted structural system consists of HE200A sections for the columns and IPE360 sections for the rafters. This choice ensures that the frame satisfies all relevant design requirements under the considered loading conditions [52, 53, 54], while maintaining adequate structural performance in terms of both resistance and stiffness.

Afterwards, once the design of the frame had been completed, a linearized buckling analysis [51] of the portal frame was performed, resulting in a critical buckling load for the first eigenmode equal to 2663 kN. The influence on the column buckling resistance (critical buckling load) of the aforementioned steel frame is investigated with respect to the following parameters:

a. Beam-to-column stiffness ratio: variation of the beam (rafter) cross-section while keeping the column section constant (HE200A).

b. Beam-to-column stiffness ratio: variation of the column cross-section while keeping the beam section constant (IPE360O).

c. Rafter inclination angle.

d. Semi-rigid behavior of the beam–column joint.

The parameters kept constant throughout the parametric study are the frame height (h) and span length (L), with h = 5 m and L = 10 m (Figure 2).

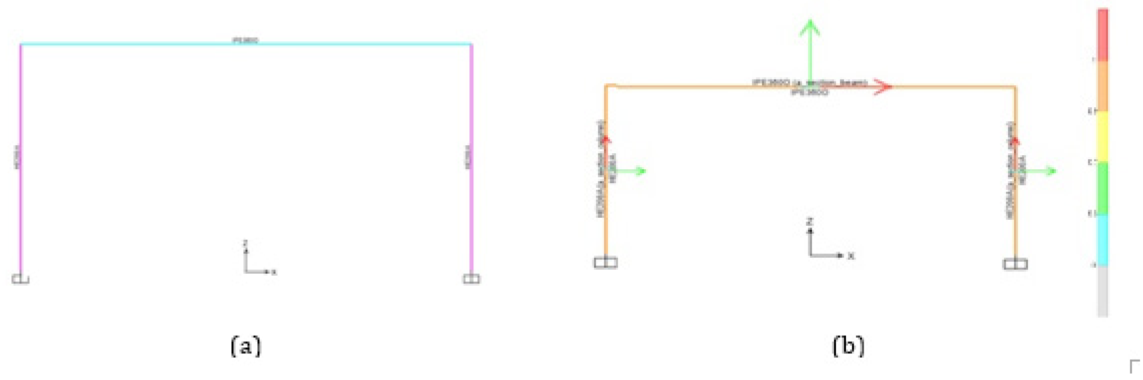

Figure 2: a) Steel structural frame, b) Utilization factors of the sections for the steel frame.

Investigation of the influence of the Beam-to-Column Stiffness ratio on the Buckling Resistance of the Steel Frame

Investigation of the influence of the beam-to-column stiffness ratio on the buckling resistance of the steel frame. Variation of the beam (rafter) cross-section while keeping the column section constant.

The cross-section of the column was kept constant as HE200A, while the beam cross-section (IPE series sections) was varied so that the ratio of the beam-to-column moments of inertia (I_2/I_1) ranged from 0 to a very large value (38). Obviously, the extreme values of this ratio are beyond practical engineering design; however, they illustrate the influence of the beam-to-column stiffness ratio on the buckling resistance of the frame columns. Subsequently, a linearized buckling analysis was carried out, and the critical buckling load ( ) was calculated. This value was normalized (non-dimensionalized) with respect to the Euler buckling load, considering the HE200A column as a pinned-pinned member (= 1107,61 kN). The results of the buckling analyses are presented both in tabular form and in graphical form (Table 2, Figure 3).

Table 2: Investigation of the influence of the beam-to-column stiffness ratio on the buckling resistance of the steel frame - variation of the beam (rafter) cross-section while keeping the column section constant.

Beam Section IPE (I2)	Beam Inertia, I2 (m4)	Column Section HE200A I1 (m4)	I2/I1	P_{cr} kN)	$P_{cr}/P_{cr,E}$
100	1,71E-06	3,69E-05	0,046316	84,964,355	0,77
120	3,11E-06	3,69E-05	0,084182	91,764,759	0,83
140	5,41E-06	3,69E-05	0,146533	101,454,527	0,92
160	8,69E-06	3,69E-05	0,235455	114,452,471	1,03
180	1,32E-05	3,69E-05	0,356717	130,086,031	1,17
180O	1,51E-05	3,69E-05	0,407638	136,020,289	1,23
180 R	1,55E-05	3,69E-05	0,420910	137,517,618	1,24
200	1,94E-05	3,69E-05	0,526273	148,511,525	1,34
200 O	2,21E-05	3,69E-05	0,598862	155,396,829	1,40
200R	2,36E-05	3,69E-05	0,640033	159,069,891	1,44
220	2,77E-05	3,69E-05	0,750813	168,114,023	1,52
220O	3,13E-05	3,69E-05	0,848862	175,347,063	1,58
220 R	3,47E-05	3,69E-05	0,940953	181,491,812	1,64
240	3,89E-05	3,69E-05	1,054,171	188,308,256	1,70
240O	4,37E-05	3,69E-05	1,183,369	195,324,082	1,76
240 R	4,82E-05	3,69E-05	1,306,338	201,276,722	1,82
270	5,79E-05	3,69E-05	1,568,256	211,980,483	1,91
270 O	6,95E-05	3,69E-05	1,881,636	222,313,802	2,01
270 R	7,31E-05	3,69E-05	1,980,498	225,113,170	2,03
300	8,36E-05	3,69E-05	2,263,272	232,113,300	2,10

300 O	9,99E-05	3,69E-05	2,706,934	240,964,276	2,18
300 R	1,05E-04	3,69E-05	2,843,987	243,290,742	2,20
330	1,18E-04	3,69E-05	3,187,974	248,316,871	2,24
330 O	1,39E-04	3,69E-05	3,767,606	255,211,022	2,30
330 R	1,47E-04	3,69E-05	3,978,873	257,333,311	2,32
360	1,63E-04	3,69E-05	4,406,826	260,976,351	2,36
360 O	1,91E-04	3,69E-05	5,159,805	266,258,913	2,40
360 R	2,03E-04	3,69E-05	5,495,666	268,216,262	2,42
400	2,31E-04	3,69E-05	6,264,897	271,884,022	2,45
400 O	2,68E-04	3,69E-05	7,245,395	275,632,847	2,49
400 R	2,89E-04	3,69E-05	7,816,901	277,435,454	2,50
400 V	3,01E-04	3,69E-05	8,163,597	278,391,767	2,51
450	3,37E-04	3,69E-05	9,138,678	280,707,007	2,53
450 O	4,09E-04	3,69E-05	11,083,424	284,275,110	2,57
450 R	4,24E-04	3,69E-05	11,484,290	284,869,277	2,57
450 V	4,62E-04	3,69E-05	12,513,543	286,238,257	2,58
500	4,82E-04	3,69E-05	13,055,255	286,813,777	2,59
500 O	5,78E-04	3,69E-05	15,650,054	289,282,654	2,61
500 R	5,99E-04	3,69E-05	16,232,394	289,738,887	2,62
550	6,71E-04	3,69E-05	18,179,848	290,989,241	2,63
500V	7,07E-04	3,69E-05	19,154,930	291,589,823	2,63
550O	7,92E-04	3,69E-05	21,440,953	292,660,814	2,64
550R	8,66E-04	3,69E-05	23,456,121	293,475,722	2,65
600	9,21E-04	3,69E-05	24,940,412	293,948,120	2,65
550V	1,02E-03	3,69E-05	27,708,559	294,818,584	2,66
600R	1,10E-03	3,69E-05	29,875,406	295,304,870	2,67
600 O	1,18E-03	3,69E-05	32,042,254	295,772,899	2,67
600 V	1,42E-03	3,69E-05	38,353,196	296,842,428	2,68

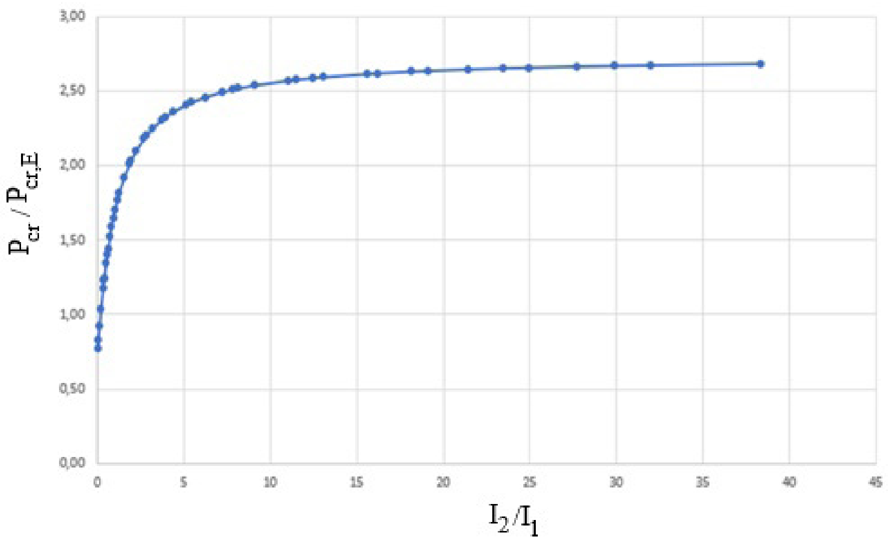

Figure 3: Investigation of the influence of the beam-to-column stiffness ratio on the buckling resistance of the steel frame - variation of the beam (rafter) cross-section while keeping the column section constant.

It is evident that augmenting the moment of inertia of the beam section, and so enhancing the stiffness of the upper joint of the column, results in an elevation of the critical buckling load of the column. For a diminutive beam section, such as IPE100, the rigidity of the upper joint is much inferior to that of the column. Consequently, the column demonstrates behavior akin to that of a cantilever member. The critical buckling load of a cantilever HE200A section, depicted in Figure 3, is 766 kN, while for the steel frame structure illustrated in Figure 4, the comparable load is 850 kN.

In constructions where the beam-to-column moment of inertia ratio is 10 or greater, corresponding to beam sections equal to or exceeding IPE450, the critical buckling load value tends to stabilize. This transpires because, beyond a specific threshold, the flexural stiffness of the beam (i.e., the moment of inertia of the beam section) becomes significantly greater than that of the original column, leading the structural behavior to resemble that of a fixed-fixed member (Figures 4 & 5).

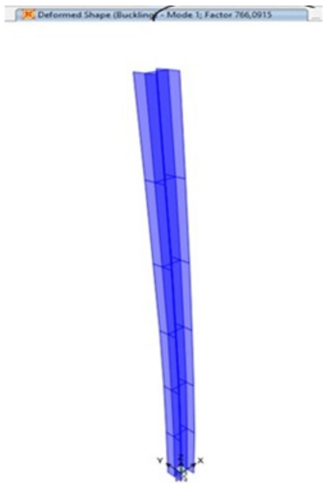

Figure 4: Critical buckling load of an HE200A cantilever section.

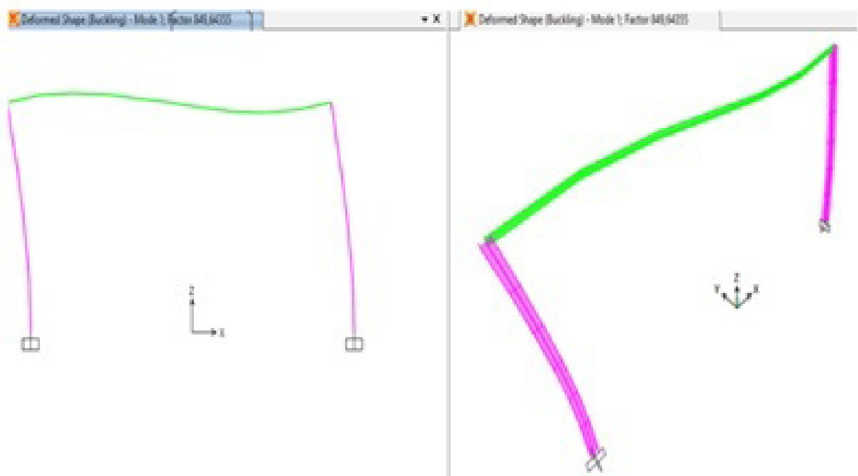

Figure 5: Critical buckling load of the frame for I_2/I_1 greater than or equal to 10.

Investigation of the influence of the beam-to-column stiffness ratio on the buckling resistance of the steel frame. Variation of the column cross-section while keeping the beam section constant

In the present parametric analyses, the beam section was kept constant, while variations were introduced to the column sections. The beam section corresponds to that of the initial frame, namely IPE 360. Similarly, the normalization of the critical loads obtained from the linearized buckling analysis was carried out using the Euler buckling load, considering the HE200A column section as a pinned-pinned member (= 1107,61 kN). The complete set of sections used for the columns belonged to the HEA and HEB series.

As a result, forty (40) different combinations of member sections and frame configurations were examined, for which the critical buckling load was calculated. The sections considered, along with the detailed analysis results, are presented in the following table (Table 3). As in the previous investigation, some combinations of member sections have no practical engineering application for the reasons mentioned earlier. Nevertheless, their study also leads to important conclusions regarding the structural behavior of the frame (Table 3, Figure 6).

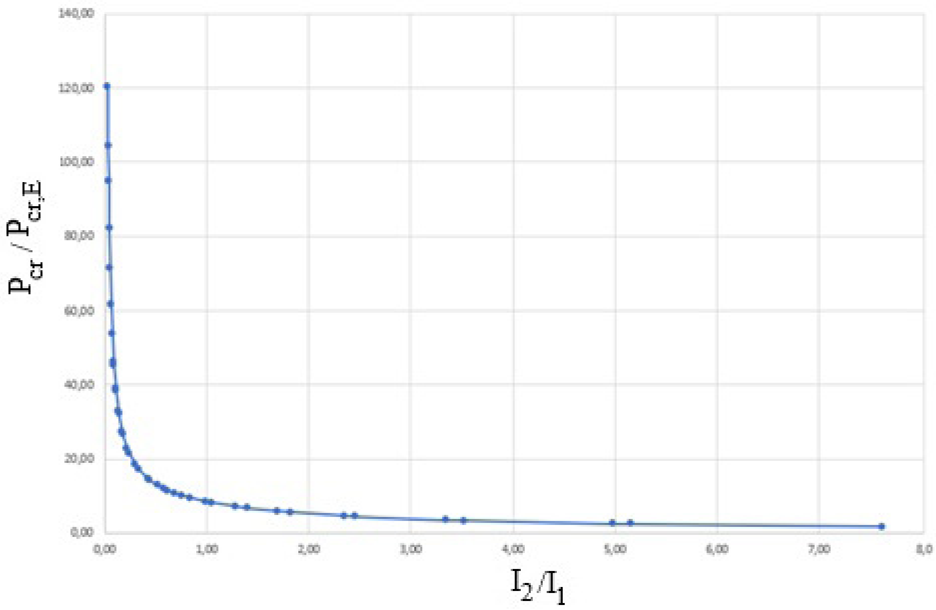

Figure 6: Investigation of the influence of the beam-to-column stiffness ratio on the buckling resistance of the steel frame - variation of the column cross-section while keeping the beam section constant

Table 3: Investigation of the influence of the beam-to-column stiffness ratio on the buckling resistance of the steel frame - variation of the column cross-section while keeping the beam section constant.

Column Section HEA-HEB (I_1)	Beam Section, IPE3600 I_2 (m^4)	Column Inertia I_1 (m^4)	I_2/I_1	P_{cr} (kN)	$P_{cr}/P_{cr,E}$
HE1000B	1,91E-04	6,45E-03	0,029549	133482,5249	120,51
HE1000A	1,91E-04	5,54E-03	0,034399	115773,0039	104,53
HE900B	1,91E-04	4,94E-03	0,038555	105120,0378	94,91
HE900A	1,91E-04	4,22E-03	0,045131	90955,00744	82,12
HE800B	1,91E-04	3,59E-03	0,053049	79279,05673	71,58
HE800A	1,91E-04	3,03E-03	0,062788	68198,17799	61,57
HE700B	1,91E-04	2,57E-03	0,074153	59507,55838	53,73
HE700A	1,91E-04	2,15E-03	0,088481	51131,75991	46,16
HE650B	1,91E-04	2,11E-03	0,090456	50343,88737	45,45
HE650A	1,91E-04	1,75E-03	0,108733	43153,60196	38,96
HE600B	1,91E-04	1,71E-03	0,111404	42509,50529	38,38
HE600A	1,91E-04	1,41E-03	0,134915	36392,20453	32,86
HE550B	1,91E-04	1,37E-03	0,139356	35645,80595	32,18
HE550A	1,91E-04	1,12E-03	0,170241	30481,70814	27,52
HE500B	1,91E-04	1,07E-03	0,177705	29652,22936	26,77

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HE500A	1,91E-04	8,70E-04	0,219041	25350,33097	22,89
HE450B	1,91E-04	7,99E-04	0,238453	23985,94537	21,66
HE450A	1,91E-04	6,37E-04	0,298964	20423,67613	18,44
HE400B	1,91E-04	5,77E-04	0,330270	19188,63434	17,32
HE400A	1,91E-04	4,51E-04	0,422676	16239,64383	14,66
HE360B	1,91E-04	4,32E-04	0,441074	15841,09927	14,30
HE340B	1,91E-04	3,67E-04	0,519640	14245,23157	12,86
HE360A	1,91E-04	3,31E-04	0,575703	13286,61670	12,00
HE320B	1,91E-04	3,08E-04	0,618105	12741,12010	11,50
HE340A	1,91E-04	2,77E-04	0,687974	11847,14536	10,70
HE300B	1,91E-04	2,52E-04	0,756853	11188,24875	10,10
HE320A	1,91E-04	2,29E-04	0,830789	10484,73522	9,47
HE280B	1,91E-04	1,93E-04	0,988583	9409,86767	8,50
HE300A	1,91E-04	1,83E-04	1,043264	9028,95881	8,15
HE260B	1,91E-04	1,49E-04	1,276810	7924,66590	7,15
HE280A	1,91E-04	1,37E-04	1,393563	7421,85465	6,70
HE240B	1,91E-04	1,13E-04	1,691829	6513,40032	5,88
HE260A	1,91E-04	1,05E-04	1,822967	6126,72474	5,53
HE220B	1,91E-04	8,09E-05	2,354468	5093,64270	4,60
HE240A	1,91E-04	7,76E-05	2,453948	4908,91802	4,43
HE200B	1,91E-04	5,70E-05	3,344452	3855,85072	3,48
HE220A	1,91E-04	5,41E-05	3,521257	3679,51785	3,32
HE180B	1,91E-04	3,83E-05	4,973890	2760,88010	2,49
HE200A	1,91E-04	3,69E-05	5,159805	2662,58913	2,40
HE180A	1,91E-04	2,51E-05	7,589641	1890,18719	1,71

It is observed that a reduction in the column cross-section, and consequently an increase in the beam-to-column moment of inertia ratio, leads to a decrease in the critical buckling load of the column, since the influence of the column stiffness plays a crucial role in its load-carrying capacity.

For small column moments of inertia (beam-to-column stiffness ratios greater than 5.5), the critical buckling load is observed to tend toward a plateau. This occurs because, below a certain column section size (approximately HE240A), the beam effectively behaves as a fixed support. As a result, further reductions in column moment of inertia no longer significantly affect the critical load, which becomes almost constant. In practical engineering structures, such large disparities between beam and column stiffness are not typically encountered.

Conversely, for values where the beam-to-column moment of inertia ratio approaches zero—i.e., for column sections larger than HE700A—the columns tend to behave like cantilevers, since the beam does not provide sufficient stiffness to restrain the upper joint of the column. In this case, the critical buckling load depends almost entirely on the column section properties.

Investigation of the influence of semi-rigid connections on the buckling resistance of the steel frame.

Idealized joint behavior is commonly assumed to fall into two extreme categories: either fully rigid connections, which are able to develop bending moments without relative rotation between the connected members, or pinned connections, which allow free rotation without developing any moment.

In real structural systems, however, such idealized behavior is not always achievable. Joints typically exhibit a reduced rotational stiffness, meaning that they resist rotation to a certain extent while simultaneously allowing partial rotation. As a result, they develop both bending moments and relative rotations, and are therefore classified as semi-rigid connections [13, 24, 31, 32]. The simulation of semi-rigid joints is carried out using a rotational spring placed at the connection point between the structural members.

The influence of beam-to-column joint stiffness is investigated for different values of rotational spring stiffness. For each case, the corresponding structural model is solved and the critical buckling load is extracted. The rotational stiffness of the spring is non-dimensional zed by dividing it by the ratio EI/h, where E is the modulus of elasticity of the column material, I is the column moment of inertia, and h is the frame height (EI/h = 561,12 kNm). As in previous cases, the critical load is normalized using the Euler buckling load of a pinned-pinned column with the same cross-section (Figure 7, Table 4).

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Table 4: Critical buckling load P_{cr} and normalized buckling load $P_{cr}/P_{cr,E}$, for various rotational stiffness values of the semi-rigid beam-to-column joint.

k_{φ} (kNm/rad)	$k_{\varphi}/(EI/h)$	P_{cr} (kN)	$P_{cr}/P_{cr,E}$
0	0	766,09150	0,69166397
10	1,78E-02	770,08013	0,69526510
100	1,78E-01	805,35423	0,72711224
1000	1,78E+00	1104,35492	0,99706433
10000	1,78E+01	2103,43062	1,89907756
100000	1,78E+02	2590,22913	2,33858249
1E+06	1,78E+03	2655,15028	2,39719640
1E+07	1,78E+04	2621,84318	2,36712516
1E+08	1,78E+05	2662,51452	2,40384519
1E+09	1,78E+06	2662,58167	2,40390582
1E+10	1,78E+07	2662,58839	2,40391188
1E+11	1,78E+08	2662,58906	2,40391249
1E+12	1,78E+09	2662,58913	2,40391255
1E+13	1,78E+10	2662,58913	2,40391255
1E+14	1,78E+11	2662,58913	2,40391255
1E+15	1,78E+12	2662,58913	2,40391255
1E+16	1,78E+13	2662,58913	2,40391255

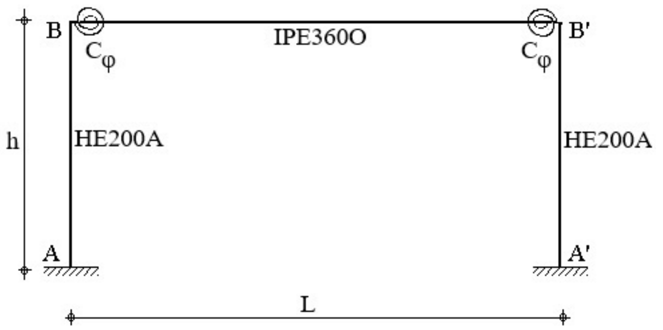

Figure 7: Steel frame with semi-rigid connections

It is observed that rotational spring stiffness values in the range of 0–1000 kNm/rad have a relatively limited effect on the critical buckling load, since the load increases only gradually, with a rate of change not exceeding 5%. In this case, the rotational stiffness is sufficiently low for the joint to behave essentially as a pinned connection, and therefore the variation in buckling resistance is not significant.

On the other hand, rotational stiffness values greater than 10^5 kNm/rad do not significantly affect the critical buckling load of the frame. This is because beyond this threshold the stiffness is sufficiently high for the joint to behave as a rigid connection. As a result, the critical load approaches the value corresponding to the original frame with fully rigid joints.

For intermediate values of rotational stiffness, a pronounced increase in the critical buckling load is observed. More specifically, for stiffness values ranging from 10 to 100 kNm/rad, the increase in load is approximately 37%, while for values between 100 and 1000 kNm/rad the increase reaches 90%. For stiffness values between 1000 and 10000 kNm/rad, an additional increase of 27% is observed. Within these ranges, the joints exhibit semi-rigid behavior (Figure 8).

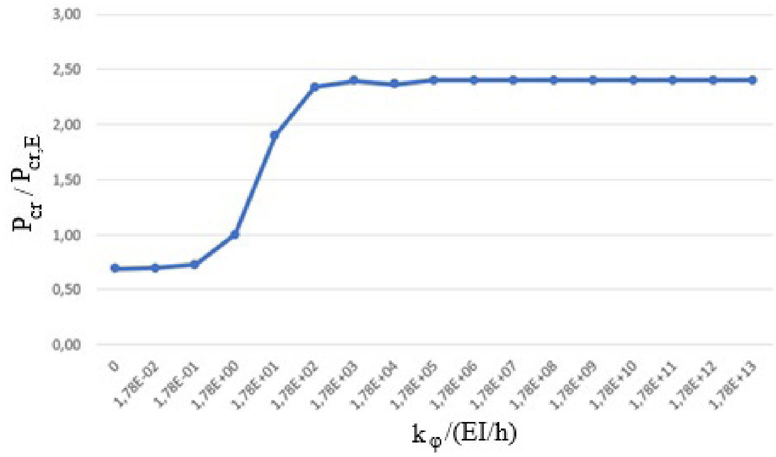

Figure 8: Normalized buckling load $P_{cr}/P_{cr,E}$, for various normalized rotational stiffness values of the semi-rigid beam-to-column joint.

Conclusions

The influence of the beam-to-column stiffness ratio and the stiffness of a semi-rigid beam-to-column connection on the buckling resistance of a planar steel frame is investigated. The steel frame represents a part of an industrial steel shed. Initially, the structure was designed for the relevant actions according to Eurocodes 0, 1, 3, and 8.

Subsequently, parametric linear buckling analyses were performed for different beam-to-column stiffness ratios. In the first set of analyses, the column section was kept constant while the beam section was varied; in the second set, the beam section was kept constant while the column section was varied. In the first case, increasing the beam section, and therefore increasing its moment of inertia, leads to an increase in the critical buckling load. For very small beam sections (e.g., IPE100), the stiffness of the upper column joint is very low compared to that of the column itself. As a result, the column behaves similarly to a cantilever member. For structures with a beam-to-column inertia ratio equal to or greater than 10, the critical buckling load tends to stabilize. This occurs because, beyond a certain threshold, the flexural stiffness of the beam becomes sufficiently large relative to the column stiffness, resulting in a structural response similar to that of a fixed-fixed column. The influence on the critical buckling load is more pronounced when the column section is varied.

In addition, a parametric investigation was carried out regarding the influence of semi-rigid beam-to-column joint stiffness on the critical buckling load of the frame. A series of analyses was performed for different values of dimensionless rotational stiffness of the joint. Low values of dimensionless rotational stiffness (1.78E+00) have only a limited effect on the critical buckling load, with an increase not exceeding 5%, as the joint behaves essentially as a pinned connection. Conversely, high values greater than 1.78E+02 do not significantly affect the critical buckling load, as the joint behaves as a fully rigid connection. In this case, the buckling resistance approaches that of the original rigid-frame configuration. For intermediate stiffness values, a sharp increase in the critical load is observed. More specifically, for dimensionless stiffness values ranging from 1,78E−02 to 1,78E−01, the increase is approximately 37%, while for values between 1,78E−01 and 1,78E+00 the increase reaches 90%. For values between 1,78E+00 and 1,78E+01, an additional increase of 27% is observed. Within these ranges, the joint exhibits semi-rigid behavior.

Overall, the results of this study highlight the significant role of both member stiffness distribution and joint flexibility in governing the buckling resistance of planar steel portal frames. The parametric investigations demonstrate that the beam-to-column stiffness ratio strongly influences the critical buckling load, particularly when variations are introduced in the column section, while the effect tends to stabilize once a sufficiently high beam stiffness is reached. In addition, the stiffness of semi-rigid beam-to-column connections has been shown to substantially affect the global stability response, especially within intermediate stiffness ranges where the transition from pinned to rigid behavior is most pronounced. Outside this range, the structural response approaches the limiting cases of idealized pinned or fully rigid joints, with comparatively minor changes in buckling capacity. These findings emphasize the importance of accurately representing both member and connection stiffness in the stability design of steel portal frames, as they can significantly alter the predicted buckling resistance and overall structural performance.

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