



Fundamentals for Cracking Verification of Reinforced Concrete Structures

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Abstract

A mathematical model, named as general method, has been set up to study the cracking behaviour of a concrete tie reinforced by ordinary bars, subjected to a monotonic load. The analysis is performed considering both the crack formation stage and the stabilized cracking stage. In particular, secondary cracks occur in the stabilized cracking stage, modifying the distribution of bond stress along the transmission length. As in the stabilized cracking stage the solution of the general method requires to carry out an iterative numerical procedure, for design purposes the maximum crack width can be determined through a simplified method that allows the main aspects of the structural behaviour to be taken into account.

Introduction

Cracking in concrete structures has to be considered normal and, in general, it does not indicate a construction defect or danger when it remains within limits established by standards or contracts [1,2]. Cracks, that can already occur in fresh concrete, are due to either direct loading or restraint or imposed deformations and appear when the tensile strength of concrete is reached. Two types of cracks can be observed in structural members, that means primary cracks, that propagate inside the concrete reaching the concrete surface, and secondary cracks or Goto cracks [3], that develop around the reinforcement bars remaining mostly inside the concrete. Limiting the maximum crack width is aimed at avoiding to impair the aesthetic aspects, also in order to not generating alarmism among people, and preserving the structural durability or the sealing capacity in case of tanks or retaining structures. To this regard appropriate limits of the crack width are introduced for the various service conditions, generally not exceeding 0.4 mm. In the following, the case of cracking due to loading is examined, as it is the most frequent case, and a mathematical modelling is proposed for the calculation of the maximum crack width in service conditions [4,5].

Stages of the Cracking Behaviour and Calculation Assumptions

Primary cracks occur in the tension chord of the structural members, where the steel reinforcement is located to control their spacing and width. For the sake of modelling, the cracking analysis of the tension chord is usually performed considering an ideal RC tie with an effective area, $A_{c,ef}$. For instance, the height of the effective area of concrete can be assumed equal to $2.5 \cdot (h - d)$, being h and d the height and the depth of the structural member, respectively. Here, the concrete tie is assumed to be reinforced by n_s bars, with diameter ϕ_s and total area A_s , and subjected to an increasing axial force F_s . Three stages can be considered, that means:

1. The uncracked stage, where the tensile strength of the concrete, f_{ct} , is not reached and both concrete and steel present a linear elastic behaviour.
2. The crack formation stage, that starts when in a certain section the concrete tensile strength is overcome, and the first crack occurs. In this stage, when the axial force F_s increases, further cracks can form.

- The stabilized cracking stage, where the crack pattern is assumed as completed and no new primary cracks can occur. In this stage, however, around the primary cracks, secondary

cracks or Goto cracks [3] can form starting from the reinforcement and propagating into the concrete (Figure 1).

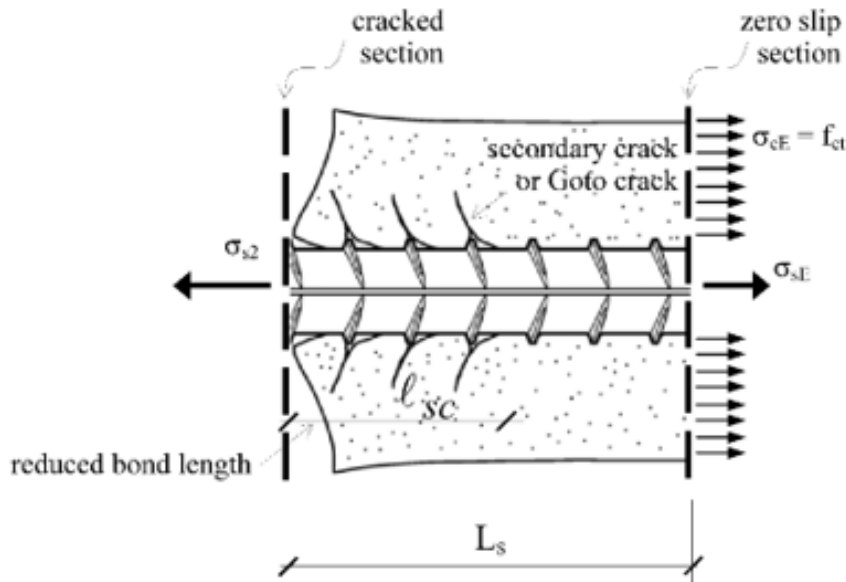


Figure 1: Stabilized cracking stage of a block of an RC tie located between the cracked section and the zero-slip section under the assumption that the crack spacing is maximum.

Because of cracking, around a single crack, during the crack formation stage, the state of stress obtained in the uncracked stage is modified, the assumption of perfect bond is lost, and a redistribution of stresses can be observed. In effects, in the crack formation stage, bond stresses, τ_{bs} , that are correlated to the slips between steel and concrete, are transmitted at the interface between steel and concrete. In this way the concrete stress, that is null at the cracked section, can increase until it reaches the concrete tensile strength, f_{ct} . Or the steel stress decreases from the cracked section to the zero-slip section. The distance, from the cracked section, where the concrete tensile strength, f_{ct} , is reached, is named transmission length, L_s . As far as the distribution of slip between steel and concrete along the transmission length, L_s , in the crack formation stage slip presents an analogous distribution of bond stress, that means slip is maximum at the cracked section and null at the zero-slip section. In effects, in the crack formation stage slips influence the bond stresses, through the bond law, along the whole transmission length. To this regard, *fib* Model Code 2010 proposes the following first ascending branch of the bond law, which is the only part of the bond law considered in service conditions:

$$\tau_{bs} = \tau_{max} \cdot \left(\frac{s_s}{s_1} \right)^\alpha \quad 0 \leq s_s \leq s_1 \quad (1)$$

where τ_{max} is the bond resistance that depends on the concrete strength, and which is equal to $k_1 \cdot \sqrt{f_{ck}}$; s_1 is the slip that corre-

sponds to the attainment of the bond resistance; α is a coefficient. The parameters k_1 , s_1 and α have tabulated values, which are valid for ribbed bars embedded in well confined concrete (in *fib* Model Code 2010: $\alpha = 0.40$; $k_1 = 2.5$ and $s_1 = 1.0$ mm in good bond condition or $k_1 = 1.25$ and $s_1 = 1.8$ mm in all other cases). Other values are given for smooth bars. From a comparison of the maximum crack width obtained with Eurocode 2, a good value of the exponent α is equal to 0,25 [4,5]. During the crack formation stage, other cracks form that correspond, locally, to the attainment of the tensile strength of concrete. But, for design purposes, the calculation of the crack width in the crack formation stage as well as in the stabilized cracking stage is made on the basis of the mean values of the material properties. Therefore, in the following it is considered as tensile strength of concrete, f_{ct} , its mean value and the transmission length, L_s , is determined accordingly. Assuming a constant value of the tensile strength, in the stabilized cracking stage the distance between two adjacent cracks, that means the crack spacing, can vary between once the transmission length, L_s , and twice the transmission length, $2 \cdot L_s$. In practice, for design purposes, it is assumed that the crack spacing is maximum and equal to twice the transmission length, as this situation, even it is an incipient condition for the formation of a new intermediate crack, represents the most unfavorable cracking condition, to which corresponds the attainment of the maximum crack width, w_{max} . Under this assumption, it results that, at the zero-slip section, the concrete stress is known and equal to

its tensile strength, f_{ct} . It is worth to note that the value obtained for the maximum crack width is a reference value determined at the concrete surface, to be used for verifications. That means, it is neither the mean value nor the characteristic value and cannot be compared with the crack width measured in situ.

General Refined Method

An RC tie is considered in the crack formation stage or in the stabilized cracking stage [4,5]. A reference system is introduced along the member axis with origin located at the zero-slip section (Figure 2).

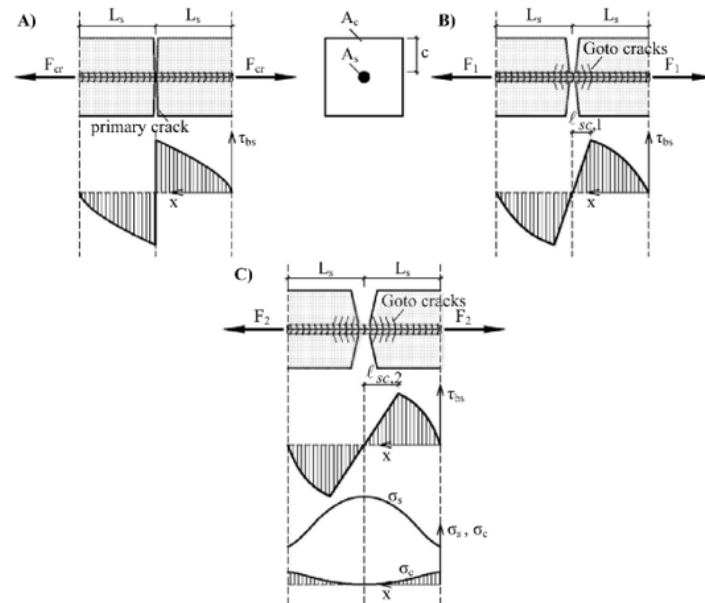


Figure 2: Cracking behaviour of an RC tie subjected to increasing forces, in the stabilized cracking stage, under the assumption that the crack spacing is maximum: A) RC tie subjected to the cracking force, that means $F_s = F_{cr}$; B) RC tie subjected to $F_s = F_1 > F_{cr}$; C) RC tie subjected to $F_s = F_2 > F_1$.

Along the transmission length, L_s , at a certain distance x from the zero-slip section, a slip between steel and concrete, $s_s(x)$, which represents the difference of the deformations of the steel, $u_s(x)$, and the concrete, $u_c(x)$, occurs:

$$s_s(x) = u_s(x) - u_c(x) \quad (2)$$

The first derivative of Eq. (2) gives the difference of the strains of the steel, ϵ_s and the concrete, ϵ_c :

$$\dot{s}_s(x) = \frac{du_s(x)}{dx} - \frac{du_c(x)}{dx} = \epsilon_s(x) - \epsilon_c(x) \quad (3)$$

while, from the second derivative of Eq. (2), it results:

$$\ddot{s}_s(x) = \frac{d\epsilon_s(x)}{dx} - \frac{d\epsilon_c(x)}{dx} \quad (4)$$

If one assumes a linear elastic behaviour for steel and concrete and considers the equilibrium of forces that act on infinitesimal blocks made of only steel or only concrete extrapolated from the RC tie, the two terms of Eq. (4) can be written in the following forms:

$$d\epsilon_s(x) = \frac{n_s \cdot \pi \cdot \phi_s \cdot \tau_{bs}(x) \cdot dx}{E_s \cdot A_s} \quad (5)$$

$$d\epsilon_c(x) = -\frac{n_s \cdot \pi \cdot \phi_s \cdot \tau_{bs}(x) \cdot dx}{E_c \cdot A_{c,ef}} \quad (6)$$

Therefore, the second order differential equation of the slipping contact between steel and concrete can be obtained, which is valid both in the crack formation stage and in the stabilized cracking stage:

$$\ddot{s}_s(x) = \frac{4 \cdot \tau_{bs}(x)}{E_s \cdot \phi_s} \cdot (1 + \alpha_e \cdot \rho_{s,ef}) \quad (7)$$

For the crack formation stage, the solution of the differential equation (7) was obtained by Balázcs in closed form [6], adopting the *fib* bond law (see Eq. (1)). On the basis of the Balázcs' solution the average bond stress, $\tau_{bs,m}$, along the transmission length can be calculated from the equilibrium condition:

$$\int_0^{L_s} n_s \cdot \pi \cdot \phi_s \cdot \tau_{bs} \cdot dx = f_{ct} \cdot A_{c,ef} = \tau_{bs,m} \cdot n_s \cdot \pi \cdot \phi_s \cdot L_s \quad (8)$$

from which, in case of $\alpha = 0.25$, it results:

$$\tau_{bs,m} = 0.414 \left(\frac{\tau_{max}^4 \cdot (1 + \alpha_e \cdot \rho_s)}{E_s \cdot s_1} \cdot f_{ct}^2 \cdot \frac{\phi_s}{\rho_s^2} \right)^{0.2} \quad (9)$$

As an alternative to Eq. (9), for concretes of characteristic compressive strength, f_{ck} , between 30 MPa and 50 MPa, it can be referred to the tabular values of Table 1, that gives the values of the average bond stress-to-mean tensile strength ratio, $\tau_{bs,m}/f_{ct}$, obtained from Eq. (9), as a function of the concrete strength, the reinforcement ratio and the bar diameter. In this way, from Eq. (8), the transmission length can be determined as:

Table 1: Values of the average bond stress-to-mean tensile strength ratio, $\tau_{bs,m}/f_{ct}$

f_{ck} (MPa)	ϕ_s (mm)	ρ_s (%)				
		1.00%	1.50%	2.00%	3.00%	5.00%
30-50	10	1.56	1.34	1.2	1.03	0.86
	25	1.88	1.61	1.44	1.24	1.03
	40	2.06	1.77	1.58	1.36	1.13

$$L_s = \frac{f_{ct} \cdot \phi_s}{4 \cdot \tau_{bs,m} \cdot \rho_{s,ef}} \quad (10)$$

In the stabilized cracking stage, it is assumed that the maximum crack spacing remains unchanged and equal to twice the transmission length. The average bond stress remains also unchanged, as it must still transmit, from the zero slip section to the cracked section, an unchanged force acting on the concrete, that means $A_{c,ef} \cdot f_{ct}$. The differential equation (7) is still valid, but the distribution of the bond stress cannot be expressed as a function of the slips along the whole transmission length without being in contrast with the equilibrium condition [4,5]. In effects, the *fib* bond law was obtained from an extensive experimental campaign based on pull-out tests in which secondary cracks cannot occur owing to the presence of concrete compressive stresses. It is, therefore, necessary to divide the transmission length into two zones (Figures 2B & 2C). The zone close to the zero-slip section is not affected by secondary cracks and, therefore, the *fib* bond law can be applied. In this zone, starting from the zero-slip section, the bond stresses tend to increase reaching a maximum value. On the contrary, the zone close to the cracked section is influenced by secondary cracks and the bond stresses tend to decrease becoming null at the crack. Moreover, differently from the crack formation stage, in the stabilized cracking stage the differential equation (7) cannot be solved in closed form, but an iterative numerical procedure is needed. Unknown of the mathematical problem is the length of the zone influenced by secondary cracks, named, according to Leonhardt [7], as reduced bond length, ℓ_{sc} . This length ℓ_{sc} is determined in such a way to obtain a null concrete stress at the cracked section. Finally, in the stabilized cracking stage the maximum crack width can be determined as difference of the elongations of the steel and the concrete along the maximum crack spacing:

$$w_{max} = 2 \cdot \int_0^{L_s} (\varepsilon_s - \varepsilon_c) \cdot dx \quad (11)$$

Simplified Calculation Method

The general method is quite complex and is certainly not suitable for design purposes. Therefore, some simplifications are introduced here. Eq. (11) can be written with reference to the average values of the steel and concrete strains, ε_{sm} and ε_{cm} , respectively, along the transmission length, L_s :

$$w_{max} = 2 \cdot L_s \cdot (\varepsilon_{sm} - \varepsilon_{cm}) \quad (12)$$

In Eq. (12) the average strains of steel and concrete can be determined from the local strains. In effects, the steel strain varies from the value $\varepsilon_{s2} = \frac{F_s}{E_s \cdot A_s}$ at the cracked section to the value

$\varepsilon_{sE} = \frac{F_s - A_{c,ef} \cdot f_{ct}}{E_s \cdot A_s} = \varepsilon_{s2} - \frac{f_{ct}}{E_s \cdot \rho_{s,ef}}$ at the zero-slip section, while the concrete strain is null at the cracked section and equal to $\varepsilon_{cE} = \frac{f_{ct}}{E_c}$ at the zero-slip section. Therefore, approximating the distributions of the steel and concrete strains (Figure 2C) with linear laws it results:

$$\varepsilon_{sm} = \varepsilon_{s2} - \frac{f_{ct}}{2 \cdot E_s \cdot \rho_{s,ef}} \quad (13)$$

$$\varepsilon_{cm} = \frac{f_{ct}}{2 \cdot E_c} \quad (14)$$

In order to take into account the effect of shrinkage of concrete, the concrete strain due to shrinkage, $\varepsilon_{c,sh}$, can be considered, leading to the following formula for the calculation of the maximum crack width:

$$w_{max} = 2 \cdot L_s \cdot (\varepsilon_{sm} - \varepsilon_{cm} - \varepsilon_{c,sh}) \quad (15)$$

Finally, the reference value of the maximum crack width is determined at the concrete surface of the structural member, not at

the reinforcement level. For this reason, it is necessary to take into account the effect of the concrete cover, c . To this regard, as shown in Figures 2A, 2B & 2C, passing from the reinforcement level to the concrete surface, the crack width tends to widen. And this tendency increases with the increase of the steel stress. It results an increase of the maximum crack width dependent on concrete cover:

$$\Delta w = \left(\frac{\sigma_{s2}}{\sigma_{sr2}} - 1 \right) \cdot \beta \cdot c \quad (16)$$

where β can be determined experimentally and assumed equal from 0.002 to 0.004, while σ_{sr2} is the steel stress at cracked section under the cracking force. In conclusion, on the basis of Eqs (15),(9),(13),(14),(16) the maximum crack width can be determined with the following formula:

$$w_{\max} = \frac{f_{ct} \cdot \phi_s}{2 \cdot \rho_s \cdot \tau_{bs,m}} \cdot (\varepsilon_{sm} - \varepsilon_{cm} - \varepsilon_{c,sh}) + \left(\frac{\sigma_{s2}}{\sigma_{sr2}} - 1 \right) \cdot \beta \cdot c \quad (17)$$

Conclusion

The calculation of the maximum crack width is usually referred to the tension chord and the effective area of concrete. Taking into account the constitutive laws of materials and the main physical aspects, in particular those relating to cracking, a mathematical general method has been set up which allows the cracking behaviour to be analysed. However, the general method presents some calculation difficulties that are not suitable for design purposes. A simplified calculation method is, therefore, proposed which allows the

main aspects of structural behaviour to be taken into account. In service conditions, the maximum crack width represents the reference value to be compared with the limits required by standards or contracts.

Acknowledgement

To God.

Conflicts of Interest

No conflict of interest.

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