ISSN: 2641-6336
DOI: 10.33552/ABBA.2026.07.000659
Annals of
Biostatistics & Biometric Applications
Iris Publishers

Review Article

Evaluating Model Performance: A Comparative Study in the Presence of Missing Data

Annastecia C Egwuekwe[1*], Kelechukwu CN Dozie[1], Stephen O Ihekun[1], Glory C Nwagwu[2] and Celestine A Igbo[3]

[1]Department of Statistics, Imo State University, Owerri, Imo State, Nigeria

[2]Department of Statistics, Kingsley Ozumba Mbadiwe University, Ideato, Imo State, Nigeria

[3]Department of Mathematics and Statistics, Federal Polytechnic Nekede, Owerri, Imo State, Nigeria

***Corresponding author:** Annastecia C Egwuekwe, Department of Statistics, Imo State University, Owerri, Imo State, Nigeria

Received Date: April 14, 2026

Published Date: May 12, 2026

Abstract

This paper discusses comparison of time series models in the presence and absence of missing data. The method adopted in this study is the Buys-Ballot procedure developed to, among other things, choice of appropriate model for decomposition of any study series, estimation of trend parameters and seasonal indices based on row, column and overall means and variances. When the models With and Without the missing observation of the transformed data are estimated and compared. Result shows that, the model of the data changed after the missing observation is estimated. The implication is that the missing observations do not only affect the seasonal indices of the data but they also affect the model of the data.

Keywords: Buys-ballot table; transformed series; missing data; trend parameter; seasonal indices; choice of model

Introduction

One of the greatest problems in descriptive time series analysis is choice of appropriate model for decomposition of any study series. That is, when to choose any of the additive, multiplicative or mixed model is not known. It is certain that the wrong use of a model will bring unreliable estimate of the component. In time series, it is assumed that the data consist of observations made sequentially in time; a systematic pattern (usually a set of identifiable components) and random noise (error). The systematic pattern includes the trend (denoted as T_t), seasonal (denoted as S_t) and the cyclical (denoted as C_t) components. The random noise (or error, irregular component) is denoted as I_t or e_t, where t stands for the particular point in time. These four classes of time series components may or may not coexist in real-life data. These components can adopt different specific functional relationship. They can be combined in an additive (additive seasonality) or a multiplicative (multiplicative seasonality) fashion and can as well take other forms such as pseudo-additive/mixed (combining the elements of both the additive and multiplicative models) model. The Additive model, Multiplicative model and Pseudo-Additive/Mixed Model are given in Equations (1) - (3) respectively:

$$\text{Additive model}: \quad X_t = T_t + S_t + C_t + I_t \quad (1)$$

$$\text{Multiplicative model}: \quad X_t = T_t \times S_t \times C_t \times I_t \quad (2)$$

$$\text{Mixed model}: \quad X_t = T_t \times S_t \times C_t + I_t \quad (3)$$

Cyclical variation which refers to the long-term oscillation or swings about the trend appears to an appreciable magnitude only in long period sets of data. However, if short period of time is involved, the cyclical component is superimposed into the trend Chatfield [1].

$$\text{Additive model}: \quad X_t = M_t + S_t + I_t \quad (4)$$
$$\text{Multiplicative model}: \quad X_t = M_t \times S_t \times I_t \quad (5)$$
$$\text{Mixed model}: \quad X_t = M_t \times S_t + I_t \quad (6)$$

In this case equation (4) - (6), can respectively, be written as:

where M_t is the trend-cycle component, S_t is the seasonal component with the property $S_{(i-1)s+j} = S_j, i = 1, 2, \ldots, m$ and is the irregular component? For equation (4), it is convenient to make assumption that the sum of the seasonal component over a complete period is

zero, ie., $\sum_{j=1}^{s} S_{t+j} = 0$ (7)

Similarly, for equations (5) and (6), the convenient variant assumption is the sum of the seasonal component over a complete period is s.

$$\sum_{j=1}^{s} S_{t+j} = s \quad (8)$$

It is equally assumed that the irregular component I_t is the Gaussian $N(0,\sigma_1^2)$ white noise for equations (4) and (6), while for equation (5), I_t is the Gaussian $N(0,\sigma_2^2)$ white noise and that $Cov(I_t, I_{t+k}) = 0, \forall k \neq 0$.

On the most appropriate condition to use any of three models, many scholars have proposed different approaches. Chatfield [1] proposed the use of the run sequence plot (time plot) to choose between additive and multiplicative models. However, they did not provide any statistical test to justify the use. Iwueze and Nwogu [2] proposed the framework for choice of model and detection of seasonal indices in time series, showed that when the trend cycle component is linear, the seasonal indices for the multiplicative model. Therefore, choice between additive and multiplicative models reduces to test for constant variance which can be used to identify the additive model. Therefore, they suggested that any test of constant variance can be used to identify the test that admits the additive model. This is an improvement over what is in existence. However, this approach can only identify the additive model when the column variance is constant, but does not tell the analyst the alternative model when the variance is not constant. Linde [3] stated that the seasonal variation is independent of the absolute level of the time series and its amplitude is relatively close for additive model.

While in a multiplicative model, the amplitude of the seasonal factor varies with level of the time series. In an additive model, the seasonal effect is the same in the same period over different years. When the seasonal effect is a proportion of the underlying trend, the multiplicative model is used. No statistical test was provided for the choice. Justo and Rivera [4] recommended the use of the coefficient of variation of seasonal quotient and differences. The seasonal differences were computed by taking the difference between a certain season of a year and the same season from the year before while the seasonal quotient was computed as the quotient of a certain season of a year and the same from the year before. Additive model is appropriate if the absolute value of coefficient of variation of the seasonal quotient is less than the absolute value of the coefficient of variation of the seasonal differences. Otherwise, it is multiplicative model. Nwogu et al [5] and Dozie, et al [6] established Chi-Square test based on the seasonal variances of the Buys-Ballot table.

The test has been theoretically verified to be quite successful and efficient for choice between mixed and multiplicative models in time series analysis. Iwueze et al [7] summarized the Buys-Ballot procedure called the Buys-Ballot table. A Buys-Ballot table summarizes data to show seasonal variations. Each line in the table is one period (usually a year) and each column is a season of the period/year (4 quarters, 12 months, etc.). A cell (i, j), of this table contain the mean value of all observations made during the period i at the season j. To analyze the data, it is helpful to include the period and seasonal totals $(T_i \text{ and } T_j)$, period and seasonal average $(\bar{X}_{i.} \text{ and } \bar{X}_{.j})$ period and seasonal standard deviations $(\hat{\sigma}_{i.} \text{ and } \hat{\sigma}_{.j})$, as part of the Buys-Ballot table. Also included for purposes of analysis are the grand total $(T_{..})$, grand mean $(\bar{X}_{..})$ and pooled standard deviation $(\hat{\sigma}_{..})$ (see Table 1). According to Wei [8], the arrangement of data in this manner in table is credited to Buys-Ballot; hence, the table has been called the Buys-Ballot table in the literature.

Table 1: Buys-Ballot Table for Seasonal Time Series.

Rows(period) i	Columns(season) j								
	1	2	$\cdots$	j	$\cdots$	s	$T_{i.}$	$\bar{X}_{i.}$	$\hat{\sigma}_{i.}$
1	X_1	X_2	$\cdots$	X_j	$\cdots$	X_s	$T_{1.}$	$\bar{X}_{1.}$	$\hat{\sigma}_{1.}$

2	X_{s+1}	X_{s+2}	$\cdots$	X_{s+j}	$\cdots$	X_{2s}	$T_{2.}$	$\bar{X}_{2.}$	$\hat{\sigma}_{2.}$
3	X_{2s+1}	X_{2s+2}	$\cdots$	X_{2s+j}	$\cdots$	X_{3s}	$T_{3.}$	$\bar{X}_{3.}$	$\hat{\sigma}_{3.}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
i	$X_{(i-1)s+1}$	$X_{(i-1)s+2}$	$\cdots$	$X_{(i-1)s+j}$	$\cdots$	$X_{(i-1)s+s}$	$T_{i.}$	$\bar{X}_{i.}$	$\hat{\sigma}_{i.}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$T_{.j}$	$T_{.1}$	$T_{.2}$	$\cdots$	$T_{.j}$	$\cdots$	$T_{.s}$	$T_{..}$	-	-
$\bar{X}_{.j}$	$\bar{X}_{.1}$	$\bar{X}_{.2}$	$\cdots$	$\bar{X}_{.j}$	$\cdots$	$\bar{X}_{.s}$	-	$\bar{X}_{..}$	-
$\hat{\sigma}_{.j}$	$\hat{\sigma}_{.1}$	$\hat{\sigma}_{.2}$	$\cdots$	$\hat{\sigma}_{.j}$	$\cdots$	$\hat{\sigma}_{.s}$	-	-	$\hat{\sigma}_{..}$

Where s = number of seasons, m = number of periods and n = ms = number of observations.

Buys-Ballot is used to estimate the trend component and seasonal indices from the chosen descriptive time series model. According to Dozie [9], Buys Ballot procedure is computationally simple when compared with other descriptive methods. The values of the estimated trend parameters and seasonal indices are easily computed in the mixed model in time series. Iwueze and Nwogu [2] proposed the Buys-Ballot estimation procedure and for the periodic means $\left(\bar{X}_{i.}, i=1,2,\ldots,m\right)$ and the overall mean $\left(\bar{X}_{..}\right)$ to estimate the trend component. Seasonal means $\left(\bar{X}_{.j}, j=1,2,\ldots,s\right)$ and the overall mean are used to estimate the seasonal indices. This research is restricted to time series with quadratic trend that admits the additive model using registered monthly church marriages over a period January, 2008 to December 2024. The, ultimate objective of this study is to compare time series models in the presence and absences of missing observations. The specific objectives are to: (a) review the Buys-Ballot procedure for in time series. (b) estimate trend parameter and seasonal indices in the presence and absence of missing observations. Based on the results and recommendations are made. This work contributes to the many existing solution of the problem of choosing the appropriate model in the presence and absence of missing data in time series analysis.

Methodology

The methods adopted in this study is the Buys-Ballot procedure developed for choice of model for decomposition of any study series, estimation of trend parameters and seasonal indices and choice of appropriate transformation based on the row, column and overall means and variances, For details of Buys-Ballot procedure for time series decomposition see Wei [8], Iwueze and Nwogu [2], Nwogu et al [5], Dozie [9], Dozie and Uwaezuoke [10], Dozie et al [6], Dozie and Ijeoma [11], Dozie and Nwanya [12], Dozie and Ihekuna [13], Dozie and Ibebuogu [14], Dozie and Ibebuogu [15], Dozie and Uwaezuoke [16], Dozie and Ihekuna [17], Dozie and Ihekuna [18], Dozie [19], Dozie and Uwaezuoke [20], Dozie [21] and Iwueze and Akpanta [22].

Review of Buys-Ballot Procedure for Time Series Decomposition

Iwueze and Nwogu [2] observed that the rows (periods) and column (seasons), with m and s representing the number of periods/years and seasons/columns respectively. This two-dimensional arrangement of a series is referred as the Buys-Ballot table.

Choice of Appropriate Transformation

According to Iwueze and Akpanta [22], transformation is a mathematical operation that changes the measurement scale of a variable. Many times, series analyst assumes normality and it is well known that variance stabilization implies normality of the series. The most popular and common are the powers of transformation such as; logarithmic transformation $\left(\log_e X_t\right)$, square-root transformation $\left(\sqrt{X_t}\right)$, inverse transformation $\left(\frac{1}{X_t}\right)$, inverse square-root transformation $\left(\frac{1}{\sqrt{X_t}}\right)$, square transformation $\left(X_t^2\right)$ and inverse square transformation $\left(\frac{1}{X_t^2}\right)$.

Selecting the best transformation can be a difficult issue and the usual statistical technique used is to estimate both the transformation and required model for the transformed X_t at the same time.

Estimation of Missing Observation

Decomposing Without the Missing Value (DWMV)

This is one of the estimation methods for missing observations proposed by I.S. Iwueze et al [23]. In this method, estimates of the trend parameters and seasonal indices obtained from the remaining observations using any of the methods of time series decomposition are substituted into the expression for the missing value.

Hence, the estimates of the missing values by this method are given by:

For Additive model $\hat{X}_{(i-1)s+j} = \hat{M}_{(i-1)s+j} + \hat{S}_{(i-1)s+j}$ (9)

For multiplicative mode $\hat{X}_{(i-1)s+j} = \hat{M}_{(i-1)s+j} \times \hat{S}_{(i-1)s+j}$ (10)

The trend-cycle components of the DWMV method for the quadratic curves is

$$\hat{M}_{(i-1)s+j} = \hat{a} + \hat{b}[(i-1)s+j] + \hat{c}[(i-1)s+j]^2 \quad (11)$$

Hence, the estimates of the missing values for additive model are given by;

$$\hat{X}_{ij} = \hat{a} + \hat{b}[(i-1)s+j] + \hat{c}[(i-1)s+j]^2 + \hat{S}_j \quad (12)$$

Estimation of Trend Parameters

Iwueze and Nwogu [2] stated the estimation of the trend parameters for an additive model when trend-cycle component is quadratic as;

$$\hat{a} = a^{|} + \left(\frac{s-1}{2}\right)\hat{b} - \left(\frac{(s-1)(2s-1)}{6}\right)\hat{c} \quad (13)$$

$$\hat{b} = \frac{b^{|}}{s} + \hat{c}(s-1) \quad (14)$$

$$\hat{c} = \frac{c^{|}}{s^2} \quad (15)$$

Estimation of Seasonal Indices

Iwueze and Nwogu [2] gave the estimation of the seasonal indices for an additive model when trend-cycle component is quadratic as;

$$\hat{S}_j = \bar{X}_{.j} - d_j \quad (16)$$

Where,

$$d_j = \hat{a} + \frac{\hat{b}}{2}(n-s) + \frac{\hat{c}(n-s)(2n-s)}{6} + \left(\hat{b} + c(\hat{n}-s)\right)j + \hat{c}j^2 \quad (17)$$

Hence,

$$\hat{S}_j = \bar{X}_{.j} - \left[\hat{a} + \frac{\hat{b}}{2}(n-s) + \frac{\hat{c}(n-s)(2n-s)}{6} + \left(\hat{b} + \hat{c}(n-s)\right)j + \hat{c}j^2\right] \quad (18)$$

The Levene's Test Statistic for the Null Hypothesis

$$H_0 : \sigma_i^2 = \sigma_j^2$$

$H_1 : \sigma_i^2 \neq \sigma_j^2 \; for\ at\ least\ one\ i \neq j$ is defined as

$$W = \frac{(N-K)\sum_{i=1}^{k} N_i\left(\bar{z}_{i} - \bar{z}_{..}\right)^2}{(k-1)\sum_{i=1}^{k}\sum_{j=1}^{N_i}\left(z_{ij} - \bar{z}_i\right)^2} \quad (19)$$

where k is the number of different groups, N_i is the number of cases in the ith group, Y_{ij} is the value of the jth observation in the ith group.

z_{ij} may be defined as deviation of y_{ij} from the mean $(\bar{y}_i)$ or from the median (y_i). That is

$$z_{ij} = y_{ij} - \bar{y}_i \; or \; y_{ij} - \bar{y}_i \quad (20)$$

$\bar{z}_{i.} = \frac{1}{N_i}\sum_{j=1}^{N_i} z_{ij}$ is the mean of the z_{ij} for group i (21)

$\bar{z}_{..} = \frac{1}{N}\sum_{i=1}^{k}\sum_{j=1}^{N_i} z_{ij}$ is mean of all z_{ij}. (22)

The test statistic W approximately follows the F-distribution with $k-1 \; and \; N-K$ degree of freedom. To suit the Buys-Ballot procedure, the levene's test statistic is modified with

$N = ms, k = s \; N_i = m$ as

$$W = \frac{(ms-s)}{s-1}\left[\frac{\sum_{j=1}^{s} m\left(\bar{z}_i - \bar{z}_{..}\right)^2}{\sum_{j=1}^{s}\sum_{i=1}^{m}\left(z_{ij} - \bar{z}_{.j}\right)^2}\right] \quad (23)$$

$$= \frac{s(m-1)}{s-1}\left[\frac{m\sum_{i=1}^{s}\left(\bar{z}_{i}-\bar{z}_{..}\right)^{2}}{\sum_{i=1}^{s}\sum_{j=1}^{m}\left(z_{ij}-\bar{z}_{.j}\right)^{2}}\right] \quad (24)$$

Real Life Example

Real life data is based on monthly number of church marriages at St. Molumba's Catholic Church Owerri in Imo State for a period of seventeen (17) years (2008-2024) shown in Appendix A. The 189 observations are transformed using the power transformation $\left(X_t^{1-\beta}\right)$ and arranged in a Buys-Ballot table as monthly data $(S=12)$ and for 17years $(m=17)$ shown in Appendix B. The time plots of the actual and transformed series given in (Figures 1-3) respectively. The trend-cycle component M_t used the quadratic: $M_t = a + bt + ct^2$ with

$$\hat{a} = 1.6454, \hat{b} = 0.0032 \; and \; \hat{c} = -0.00003$$

for the transformed data with the missing values and $\hat{a} = 2.1234, \hat{b} = 0.0153 \; and \; \hat{c} = -0.0001$ for the transformed data without the missing values.

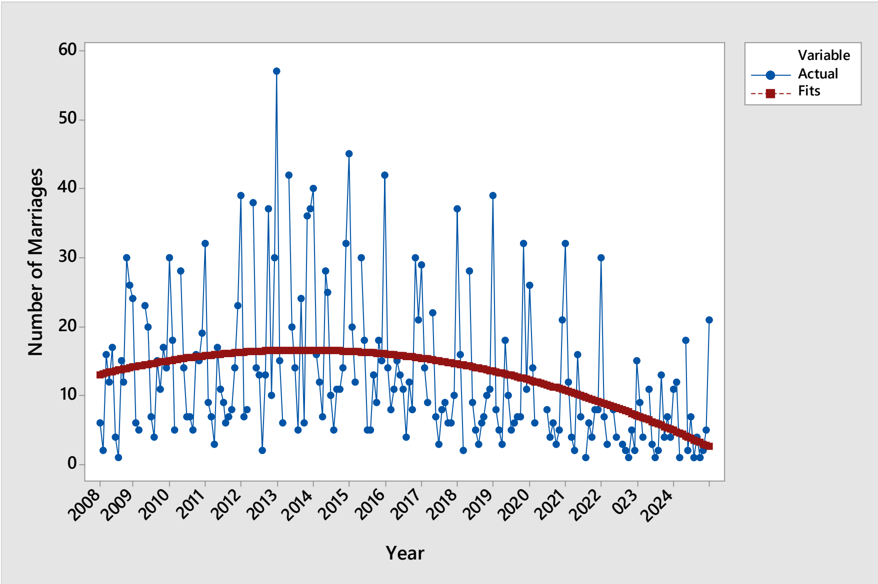

Figure 1: Time Plot of the Actual Series with Missing Observations.

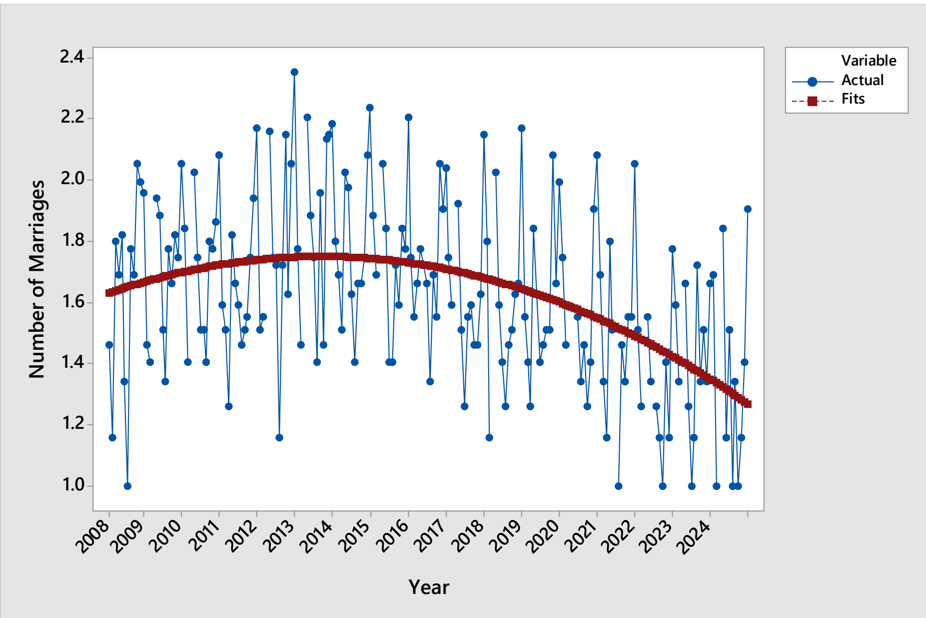

Figure 2: Transformed Series with Missing Observations.

Citation: Annastecia C Egwuekwe*, Kelechukwu CN Dozie, Stephen O Ihekun, Glory C Nwagwu and Celestine A Igbo, Evaluating Model Performance: A Comparative Study in the Presence of Missing Data. Annal Biostat & Biomed Appli. 7(2): 2026. ABBA.MS.ID.000659.
DOI: 10.33552/ABBA.2026.07.000659.

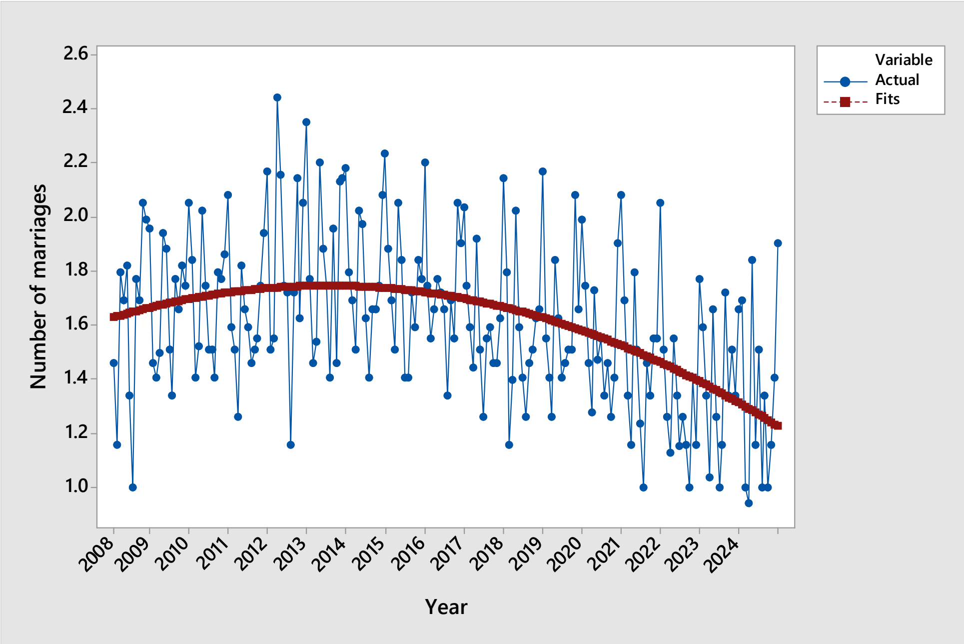

Figure 3: Transformed Series without Missing Observations.

Observations

Using (13), (14), (15) and (18)

Table 2: Seasonal Indices with Missing Values of the Transformed Data.

j	$\bar{X}_{.j}$	$\hat{S}_{.j}$	$Adj\ \hat{S}_{.j}$
1	1.6703	0.0677	0.0364
2	1.4109	-0.1895	-0.2208
3	1.4414	-0.1567	-0.1880
4	1.8957	0.2999	0.2686
5	1.6426	0.0492	0.0179
6	1.4831	-0.1079	-0.1392
7	1.2977	-0.2908	-0.3221
8	1.5832	-0.0028	-0.0341
9	1.5021	-0.0813	-0.1126
10	1.7058	0.1251	0.0938
11	1.7536	0.1756	0.1443
12	2.0622	0.4870	0.4557
$\sum_{j=1}^{12}\hat{S}_{.j}$		0.3755	0.0000

$$\hat{c} = \frac{-0.0001}{(12)^2}$$
$$= \frac{-0.0036}{144}$$
$$= -0.00003$$

$$\hat{b} = \frac{0.0424}{12} + (-0.00003)(12-1)$$
$$= \frac{0.0424}{12} - 0.00003(11)$$
$$= 0.00320$$

$$\hat{a} = 1.6265 + \left(\frac{12-1}{2}\right)(0.00320) - \left(\frac{(12-1)(2(12)-1)}{6}\right)(-0.00003)$$
$$= 1.6265 + 0.0176 + 0.00127$$
$$= 1.6454$$

$$d_j = 1.6454 + \frac{0.0032}{2}(189-12) + \frac{(-0.00003)(189-12)(2(189)-12)}{6} + (0.0032 + (-0.00003)(189-12))\,j + (-0.00003)\,j^2$$
$$= 1.6454 + 0.2832 - 0.3239 - 0.0021j - 0.00003j^2$$
$$d_j = 1.6047 - 0.0021j - 0.00003j^2$$

$$\hat{S}_j = \bar{X}_{.j} - \left[1.6047 - 0.0021j - 0.00003j^2\right]$$
$$= \bar{X}_{.j} - 1.6047 + 0.0021j + 0.00003j^2$$

Estimation of Trend Parameters and Seasonal Indices of the Transformed Data Without Missing Values

Using (13), (14), (15) and (18)

Table 3: Seasonal Indices of the Transformed Data Without Missing Values.

j	$\bar{X}_{.j}$	$\hat{S}_{.j}$	$Adj\ \hat{S}_{.j}$
1	1.6703	0.1040	0.0439
2	1.4109	-0.1529	-0.2130
3	1.3815	-0.1797	-0.2397
4	1.886	0.3275	0.2674
5	1.6327	0.0769	0.0169
6	1.4493	-0.1037	-0.1638
7	1.2977	-0.2525	-0.3125
8	1.5832	0.0360	-0.0241
9	1.5021	-0.0422	-0.1022
10	1.7058	0.1646	0.1045
11	1.7536	0.2155	0.1554
12	2.0622	0.5272	0.4672
$\sum_{j=1}^{12} \hat{S}_{.j}$		0.7206	0.0000

Citation: Annastecia C Egwuekwe*, Kelechukwu CN Dozie, Stephen O Ihekun, Glory C Nwagwu and Celestine A Igbo, Evaluating Model Performance: A Comparative Study in the Presence of Missing Data. Annal Biostat & Biomed Appli. 7(2): 2026. ABBA.MS.ID.000659.
DOI: 10.33552/ABBA.2026.07.000659.

$$\hat{c} = \frac{-0.0038}{(12)^2} = \frac{-0.0038}{144} = -0.00003$$

$$\hat{b} = \frac{0.0441}{12} + (-0.00003)(12-1) = \frac{0.0441}{12} - 0.00003(11) = 0.0033$$

$$\hat{a} = 1.6127 + \left(\frac{12-1}{2}\right)(0.0033) - \left(\frac{(12-1)(2(12)-1)}{6}\right)(-0.00003)$$
$$= 1.6127 + 0.0182 + 0.00127$$
$$= 1.6322$$

$$d_j = 1.6322 + \frac{0.0033}{2}(204-12) + \frac{(-0.00003)(204-12)(2(204)-12)}{6} + \left(0.0033 + (-0.00003)(204-12)\right)j + (-0.00003)j^2$$
$$= 1.6322 + 0.3168 - 0.38016 - 0.00246j - 0.00003j^2$$
$$d_j = 1.56884 - 0.00246j - 0.00003j^2$$

$$\hat{S}_j = \bar{X}_{.j} - \left[1.56884 - 0.00246j - 0.00003j^2\right]$$
$$= \bar{X}_{.j} - 1.56884 + 0.00246j + 0.00003j^2$$

Case 1: Test of Appropriate Model with Missing Observations

To test the appropriate model with missing observation. The first step is verifying whether the data admits the additive model. The test statistic in (24) is used. The null hypothesis that the data admits additive is rejected, if W is greater than the critical value, which

for $\alpha = 0.05$ level of significance and $m-1=19$ degrees of freedom equal to or do not reject H_0 otherwise.

From Appendix C and Table 4-7

Table 4: Transformed data With Missing observations.

	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	total	$\bar{y}_{i.}$	$\sigma_{i.}$
2008	1.4608	1.1579	1.7975	1.6914	1.8207	1.3407	1.0000	1.7731	1.6914	2.0531	1.9919	1.9585	19.7370	1.6447	0.3357
2009	1.4608	1.4055		1.9409	1.8844	1.5092	1.3407	1.7731	1.6606	1.8207	1.7475	2.0531	18.5964	1.6906	0.2343
2010	1.8429	1.4055		2.0234	1.7475	1.5092	1.5092	1.4055	1.7975	1.7731	1.8640	2.0813	18.9590	1.7235	0.2355
2011	1.5916	1.5092	1.2616	1.8207	1.6606	1.5916	1.4608	1.5092	1.5524	1.7475	1.9409	2.1703	19.8161	1.6513	0.2406
2012	1.5092	1.5524		2.1584	1.7475	1.7203	1.1579	1.7203	2.1462	1.6274	2.0531	2.3516	19.7442	1.7949	0.3493
2013	1.7731	1.4608		2.2045	1.8844	1.7475	1.4055	1.9585	1.4608	2.1338	2.1462	2.1819	20.3569	1.8506	0.3056
2014	1.7975	1.6914	1.5092	2.0234	1.9754	1.6274	1.4055	1.6606	1.6606	1.7475	2.0813	2.2369	21.4167	1.7847	0.2470
2015	1.8844	1.6914		2.0531	1.8429	1.4055	1.4055	1.7203	1.5916	1.8429	1.7731	2.2045	19.4151	1.7650	0.2447
2016	1.7475	1.5524	1.6606	1.7731	1.7203	1.6606	1.3407	1.6914	1.5524	2.0531	1.9039	2.0384	20.6944	1.7245	0.2039
2017	1.7475	1.5916		1.9227	1.5092	1.2616	1.5524	1.5916	1.4608	1.4608	1.6274	2.1462	17.8716	1.6247	0.2413
2018	1.7975	1.1579		2.0234	1.5916	1.4055	1.2616	1.4608	1.5092	1.6274	1.6606	2.1703	17.6656	1.6060	0.3038
2019	1.5524	1.4055	1.2616	1.8429	1.6274	1.4055	1.4608	1.5092	1.5092	2.0813	1.6606	1.9919	19.3081	1.6090	0.2479
2020	1.7475	1.4608				1.5524	1.3407	1.4608	1.2616	1.4055	1.9039	2.0813	14.2144	1.5794	0.2745
2021	1.6914	1.3407	1.1579	1.7975	1.5092		1.0000	1.4608	1.3407	1.5524	1.5524	2.0531	16.4561	1.4960	0.2925
2022	1.5092	1.2616		1.5524	1.3407		1.2616	1.1579	1.0000	1.4055	1.1579	1.7731	13.4198	1.3420	0.2260
2023	1.5916	1.3407		1.6606	1.2616	1.0000	1.1579	1.7203	1.3407	1.5092	1.3407	1.6606	15.5837	1.4167	0.2307
2024	1.6914	1.0000		1.8429	1.1579	1.5092	1.0000	1.3407	1.0000	1.1579	1.4055	1.9039	15.0093	1.3645	0.3367
Total	28.3959	23.9851	8.6483	30.3312	26.2810	22.2460	22.0607	26.9138	25.5355	28.9990	29.8110	35.0570	308.2645		
$\bar{y}_{.j}$	1.6703	1.4109	1.4414	1.8957	1.6426	1.4831	1.2977	1.5832	1.5021	1.7058	1.7536	2.0622		1.6310	
$\sigma_{.j}$	0.1377	0.1894	0.2548	0.1810	0.2363	0.1923	0.1787	0.1959	0.2760	0.2762	0.2763	0.1707			0.2966

Table 5: Calculation of the values of the measured variable of the Levene's test statistic; $Z_{ij} = \left| y_{ij} - \bar{y}_{,j} \right|$.

	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	total	$\bar{z}_{i.}$	$\sigma_{i.}$
2008	0.2096	0.2530	0.3561	0.2043	0.1781	0.1424	0.2977	0.1900	0.1893	0.3473	0.2383	0.1037	2.7098	0.2258	0.0770
2009	0.2096	0.0054	1.4414	0.0452	0.2418	0.0261	0.0430	0.1900	0.1585	0.1149	0.0061	0.0091	2.4910	0.2076	0.3979
2010	0.1725	0.0054	1.4414	0.1277	0.1049	0.0261	0.2115	0.1777	0.2954	0.0673	0.1105	0.0191	2.7594	0.2300	0.3910
2011	0.0788	0.0983	0.1798	0.0750	0.0180	0.1085	0.1631	0.0740	0.0503	0.0416	0.1873	0.1081	1.1828	0.0986	0.0543
2012	0.1612	0.1415	1.4414	0.2627	0.1049	0.2372	0.1398	0.1371	0.6441	0.0784	0.2995	0.2894	3.9372	0.3281	0.3815
2013	0.1028	0.0499	1.4414	0.3088	0.2418	0.2644	0.1078	0.3753	0.0413	0.4280	0.3926	0.1197	3.8739	0.3228	0.3778
2014	0.1272	0.2805	0.0678	0.1277	0.3329	0.1444	0.1078	0.0774	0.1585	0.0416	0.3277	0.1748	1.9682	0.1640	0.0986
2015	0.2140	0.2805	1.4414	0.1574	0.2003	0.0776	0.1078	0.1371	0.0895	0.1370	0.0196	0.1424	3.0045	0.2504	0.3812
2016	0.0771	0.1415	0.2192	0.1226	0.0777	0.1775	0.0430	0.1082	0.0503	0.3473	0.1503	0.0237	1.5385	0.1282	0.0900
2017	0.0771	0.1807	1.4414	0.0270	0.1334	0.2215	0.2547	0.0084	0.0413	0.2451	0.1262	0.0840	2.8408	0.2367	0.3886
2018	0.1272	0.2530	1.4414	0.1277	0.0510	0.0776	0.0361	0.1224	0.0071	0.0784	0.0930	0.1081	2.5229	0.2102	0.3926
2019	0.1179	0.0054	0.1798	0.0528	0.0151	0.0776	0.1631	0.0740	0.0071	0.3755	0.0930	0.0703	1.2317	0.1026	0.1025
2020	0.0771	0.0499	1.4414	1.8957	1.6426	0.0693	0.0430	0.1224	0.2405	0.3003	0.1503	0.0191	6.0517	0.5043	0.7083
2021	0.0211	0.0702	0.2835	0.0982	0.1334	1.4831	0.2977	0.1224	0.1614	0.1534	0.2012	0.0091	3.0345	0.2529	0.3976
2022	0.1612	0.1493	1.4414	0.3433	0.3018	1.4831	0.0361	0.4253	0.5021	0.3003	0.5957	0.2890	6.0286	0.5024	0.4737
2023	0.0788	0.0702	1.4414	0.2351	0.3810	0.4831	0.1398	0.1371	0.1614	0.1967	0.4129	0.4016	4.1390	0.3449	0.3729
2024	0.0211	0.4109	1.4414	0.0528	0.4847	0.0261	0.2977	0.2425	0.5021	0.5479	0.3481	0.1583	4.5334	0.3778	0.3836
Total	2.0341	2.4455	17.1414	4.2640	4.6434	5.1253	2.4898	2.7212	3.3002	3.8011	3.7524	2.1296	53.8479		
$\bar{z}_{.j}$	0.1197	0.1439	1.0083	0.2508	0.2731	0.3015	0.1465	0.1601	0.1941	0.2236	0.2207	0.1253		0.2640	
$\sigma_{.j}$	0.0612	0.1178	0.6069	0.4341	0.3764	0.4590	0.0956	0.1057	0.1901	0.1522	0.1567	0.1117			0.368935

Citation: Annastecia C Egwuekwe*, Kelechukwu CN Dozie, Stephen O Ihekun, Glory C Nwagwu and Celestine A Igbo, Evaluating Model Performance: A Comparative Study in the Presence of Missing Data. Annal Biostat & Biomed Appli. 7(2): 2026. ABBA.MS.ID.000659.
DOI: 10.33552/ABBA.2026.07.000659.

Table 6: Calculation of the squared deviation of the seasonal points from the seasonal mean.

$$\left(z_{ij} - \bar{z}_{.j}\right)^2$$

	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	total
2008	0.0081	0.0119	0.4253	0.0022	0.0090	0.0253	0.0229	0.0009	0.0000	0.0153	0.0003	0.0005	0.5217
2009	0.0081	0.0192	0.1875	0.0423	0.0010	0.0758	0.0107	0.0009	0.0013	0.0118	0.0461	0.0135	0.4181
2010	0.0028	0.0192	0.1875	0.0152	0.0283	0.0758	0.0042	0.0003	0.0103	0.0244	0.0122	0.0113	0.3915
2011	0.0017	0.0021	0.6864	0.0309	0.0651	0.0372	0.0003	0.0074	0.0207	0.0331	0.0011	0.0003	0.8863
2012	0.0017	0.0000	0.1875	0.0001	0.0283	0.0041	0.0000	0.0005	0.2025	0.0211	0.0062	0.0270	0.4792
2013	0.0003	0.0088	0.1875	0.0034	0.0010	0.0014	0.0015	0.0463	0.0233	0.0418	0.0296	0.0000	0.3449
2014	0.0001	0.0187	0.8846	0.0152	0.0036	0.0247	0.0015	0.0068	0.0013	0.0331	0.0115	0.0024	1.0034
2015	0.0089	0.0187	0.1875	0.0087	0.0053	0.0501	0.0015	0.0005	0.0110	0.0075	0.0405	0.0003	0.3405
2016	0.0018	0.0000	0.6227	0.0165	0.0382	0.0154	0.0107	0.0027	0.0207	0.0153	0.0050	0.0103	0.7592
2017	0.0018	0.0014	0.1875	0.0501	0.0195	0.0064	0.0117	0.0230	0.0233	0.0005	0.0089	0.0017	0.3359
2018	0.0001	0.0119	0.1875	0.0152	0.0493	0.0501	0.0122	0.0014	0.0350	0.0211	0.0163	0.0003	0.4004
2019	0.0000	0.0192	0.6864	0.0392	0.0666	0.0501	0.0003	0.0074	0.0350	0.0231	0.0163	0.0030	0.9466
2020	0.0018	0.0088	0.1875	2.7056	1.8753	0.0539	0.0107	0.0014	0.0022	0.0059	0.0050	0.0113	4.8694
2021	0.0097	0.0054	0.5254	0.0233	0.0195	1.3961	0.0229	0.0014	0.0011	0.0049	0.0004	0.0135	2.0236
2022	0.0017	0.0000	0.1875	0.0086	0.0008	1.3961	0.0122	0.0703	0.0948	0.0059	0.1406	0.0268	1.9454
2023	0.0017	0.0054	0.1875	0.0002	0.0116	0.0330	0.0000	0.0005	0.0011	0.0007	0.0369	0.0764	0.3551
2024	0.0097	0.0713	0.1875	0.0392	0.0447	0.0758	0.0229	0.0068	0.0948	0.1052	0.0162	0.0011	0.6754
Total	0.0599	0.2220	5.8939	3.0157	2.2673	3.3716	0.1461	0.1787	0.5783	0.3706	0.3929	0.1996	16.6967

Table 7: Calculation of $m\left(\bar{z}_{.j}-\bar{z}_{..}\right)^2$

$\bar{z}_{.j}$	$\bar{z}_{.}$	$\bar{z}_{.j}-\bar{z}_{.}$	$\left(\bar{z}_{.j}-\bar{z}_{..}\right)^2$	$17\times\left(\bar{z}_{.j}-\bar{z}_{..}\right)^2$
0.1197	0.264	-0.1443	0.0208	0.3540
0.1439	0.264	-0.1201	0.0144	0.2452
1.0083	0.264	0.7443	0.5540	9.4177
0.2508	0.264	-0.0132	0.0002	0.0030
0.2731	0.264	0.0091	0.0001	0.0014
0.3015	0.264	0.0375	0.0014	0.0239
0.1465	0.264	-0.1175	0.0138	0.2347
0.1601	0.264	-0.1039	0.0108	0.1835
0.1941	0.264	-0.0699	0.0049	0.0831
0.2236	0.264	-0.0404	0.0016	0.0277
0.2207	0.264	-0.0433	0.0019	0.0319
0.1253	0.264	-0.1387	0.0192	0.3270
				10.9331

where m=17.

$W=\frac{12*(17-1)(10.9331)}{(12-1)(16.6967)}=11.43$ and the critical value is

$$F_{0.05,k-1,N-k}=F_{0.05,(12-1),(204-12)}=F_{0.05,11,192}=1.82$$

$W=11.43$ when compared with the critical value (1.82). W is greater, indicating that the data does not admit the additive model.

Case 2: Test of Appropriate Model Without Missing Observations

Again, to determine test for appropriate model Without missing observation. The first step is check whether the data admits the additive model. The test statistic in (24) is used. The null hypothesis that the data admits additive is rejected, if W is greater than the critical value, which for $\alpha=0.05$ level of significance and $m-1=19$ degrees of freedom equal to or do not reject H_0 otherwise.

From Appendix D and Table 8

$W=\frac{12*(17-1)(0.2901)}{(12-1)(3.7146)}=1.36$ and the critical value

$$F_{0.05,k-1,N-k}=F_{0.05,(12-1),(204-12)}=F_{0.05,11,192}=1.82$$

$W=1.36$ when compared with the critical value (1.82). W is less than, indicating that the data admits additive model.

Comparison of Case 1 and Case 2

In comparing case 1 and case 11, while the decomposition model for case 1 is not additive in the presence of missing data. The model structure of case 11 admits additive after the estimation. We observed that the model of the data changed after the missing observation is estimated. The implication is that the missing observations do not only affect the seasonal indices of the data but they also affect the model of the data.

Summary, Conclusion and Recommendation

Summary

This study discussed the impact of missing data on quadratic trend cycle and seasonal indices. The aim of this study is to Compare the model of the data With and Without missing observations. Specific objectives are to: (1) estimate the missing observations. (2) estimate the trend parameter and seasonal indices of the data With and Without missing observations. The methods adopted in this study are: 1) the Buys-Ballot estimation procedure used for the estimation of trend parameters and seasonal indices 2) the Decomposing without the Missing Value (DWMV); used of the estimation of missing observations. 3) The Levene's test for constant variance used for the identification of appropriate model. Empirical example is taken from the Marriage register from St Molumba's Catholic Church, Owerri, Imo State on monthly records of number of marriages over a period of 2008 to 2024. The trend parameter and the seasonal indices With and Without the missing observation are estimated. Results show that, the model of the data in the presence and absence of missing observations are compared and result shows that the model of the data changed after the missing observation was estimated.

Conclusion

This study has discussed the impact of missing data on quadratic trend cycle and seasonal indices. The emphasis is to identify and compare the model of the data with and without the missing observations. The Buys-Ballot estimation procedure was used to estimates the trend parameters and seasonal Indices and the Decomposing without the Missing Value (DWMV) was adopted for the estimation of the missing values. Also, using the Levene's test for constant variance for the identification of appropriate model for the data with and without the missing observation, Results show that the missing observations do not only affect the seasonal indices of the data but they also affect the model of the data.

Recommendation

For researchers who may want to work on time series data with missing observations, we recommend that identification of the appropriate model of the data after the estimation of missing observations is necessary as wrong modeling of time series data may lead erroneous forecast. However, this study is limited to time series in which the trend-cycle component is quadratic. Therefore, cases where trend-cycle components are linear and exponential are recommended for further investigation.

References

1. Chatfield C (2004) The Analysis of Time Series: An Introduction 6th Edition, Chapman and Hall London.
2. Iwueze IS, Nwogu EC (2014) Frame Work for Choice of Models and Detection of Seasonal Effect in Time Series. Far East Journal of Theoretical Statistics 48(1): 45-66.
3. Linde P (2005) Seasonal Adjustment, Statistics Denmark.
4. Justo P, Rivera MA (2010) Descriptive Analysis of Time Series applied to housing prices in Spain, management mathematics for European Schools.
5. Nwogu EC, Iwueze IS, Dozie KCN, Mbachu HI (2019) Choice between Mixed and Multiplicative Models in Time Series Decomposition. Asian Journal of Statistics and Application 9(5): 153-159.
6. Dozie KCN, Nwogu EC, Ijomah MA (2020) Effect of Missing Observations on Buys-Ballot Estimates of Time Series Components. Asian Journal of Probability and Statistics 6(3): 13-24.
7. Iwueze IS, Nwogu EC, Ohakwe J, Ajarogu JC (2011) Uses of the Buys-Ballot table in Time Series Analysis. Applied Mathematics 2(5): 633-645.
8. Wei WWS (1989) Time Series Analysis: Univariate and Multivariate Methods, Addison-Wesley, Redwood city.
9. Dozie Kelechukwu CN (2020) Buys-Ballot Estimates for Mixed Model in Descriptive Time Series. International Journal of Theoretical and Mathematical Physics 10(1): 22-27.
10. Dozie KCN, Uwaezuoke MU (2020) Properties of Buys-Ballot Estimates for Mixed Model in Time Series Decomposition. Galore International Journal of Applied Sciences and Humanities 4(2): 35-40.
11. Dozie KCN, Ijomah MA (2020) A Comparative Study on Additive and Mixed Models in Descriptive Time Series. American Journal of Mathematical and Computer Modelling. 5(1): 12-17.
12. Dozie KCN, Nwanya JCN (2020) Comparison of Mixed and Multiplicative Models when Trend-Cycle Component is Linear. Asian Journal of Advanced Research and Reports 12(4): 32-42.
13. Dozie KCN, Ihekuna SO (2020) Buys-Ballot Estimates of Quadratic Trend Component and Seasonal Indices and Effect of Incomplete Data in Time Series. International Journal of Science and Healthcare Research 5(2): 341-348.
14. Dozie KCN, Ibebuogu CC (2020) Estimates of Time Series Components of Road Traffic Accidents and Effect of Incomplete Observations: Mixed Model CASE. International Journal of Research and Review 7(6): 343-351.
15. Dozie KCN, Ibebuogu CC (2021) Road Traffic Offences in Nigeria: An Empirical Analysis using Buys-Ballot Approach. Asian Journal of Probability and Statistics 12(1): 68-78.
16. Dozie KCN, Uwaezuoke MU (2021) Seasonal Analysis of Average Monthly Exchange Rate of Central Bank of Nigeria (CBN) when Trend Cycle Component is Quadratic. International Journal of Research and Innovation in Applied Science (IJRIAS) 6(2): 201-205.
17. Dozie KCN, Ihekuna SO (2022) Additive Seasonality in Time Series using Row and Overall Sample Variances of the Buys-Ballot Table. Asian Journal of Probability and Statistics 18(3): 1-9.
18. Dozie KCN, Ihekuna SO (2023) The Effect of Missing Data on Estimates of Exponential Trend-Cycle and Seasonal Components in Time Series: Additive Case. Asian Journal Probability and Statistics 24(1): 22-36.
19. Dozie KCN (2023) Buys-Ballot Estimates for Overall Sample Variances and Their Statistical Properties. Asian Journal of Advanced Research and Reports 17(10): 213-223.
20. Dozie KCN, Uwaezuoke MU (2023) The Proposed Buys-Ballot Estimates for Multiplicative Model with Error Variances. Journal of Engineering Research and Reports 25(8): 94-106.
21. Dozie KCN (2024) Assessing Performance of Estimation Techniques in Time Series Analysis when Trend-Cycle Component is Linear. Asian Journal of Research in Computer Science 17(12): 18-29.
22. Iwueze IS, Akpanta AC (2009) Applying the Bartlett Transformation Method to Time Series Data. Journal of Mathematical Science 20(3): 227-243.
23. Iwueze IS, Nwogu EC, Nlebedim VU, Nwosu UJ, Chinyem UE (2018) Comparison of Method of Estimating Missing Values in Time Series. Open Journal of Statistics 8(2): 390-399.

APPENDIX

Appendix A: Actual Data with Missing Observations.

	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
2008	6	2	16	12	17	4	1	15	12	30	26	24
2009	6	5		23	20	7	4	15	11	17	14	30
2010	18	5		28	14	7	7	5	16	15	19	32
2011	9	7	3	17	11	9	6	7	8	14	23	39
2012	7	8		38	14	13	2	13	37	10	30	57
2013	15	6		42	20	14	5	24	6	36	37	40
2014	16	12	7	28	25	10	5	11	11	14	32	45
2015	20	12		30	18	5	5	13	9	18	15	42
2016	14	8	11	15	13	11	4	12	8	30	21	29
2017	14	9		22	7	3	8	9	6	6	10	37
2018	16	2		28	9	5	3	6	7	10	11	39
2019	8	5	3	18	10	5	6	7	7	32	11	26
2020	14	6				8	4	6	3	5	21	32
2021	12	4	2	16	7		1	6	4	8	8	30
2022	7	3		8	4		3	2	1	5	2	15
2023	9	4		11	3	1	2	13	4	7	4	11
2024	12	1		18	2	7	1	4	1	2	5	21

Source: St Molumba's Catholic Church, Owerri, Imo State.

Appendix B: Transformed Data with Missing Observations.

	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
2008	1.4608	1.1579	1.7975	1.6914	1.8207	1.3407	1.0000	1.7731	1.6914	2.0531	1.9919	1.9585
2009	1.4608	1.4055		1.9409	1.8844	1.5092	1.3407	1.7731	1.6606	1.8207	1.7475	2.0531
2010	1.8429	1.4055		2.0234	1.7475	1.5092	1.5092	1.4055	1.7975	1.7731	1.8640	2.0813
2011	1.5916	1.5092	1.2616	1.8207	1.6606	1.5916	1.4608	1.5092	1.5524	1.7475	1.9409	2.1703
2012	1.5092	1.5524		2.1584	1.7475	1.7203	1.1579	1.7203	2.1462	1.6274	2.0531	2.3516
2013	1.7731	1.4608		2.2045	1.8844	1.7475	1.4055	1.9585	1.4608	2.1338	2.1462	2.1819
2014	1.7975	1.6914	1.5092	2.0234	1.9754	1.6274	1.4055	1.6606	1.6606	1.7475	2.0813	2.2369
2015	1.8844	1.6914		2.0531	1.8429	1.4055	1.4055	1.7203	1.5916	1.8429	1.7731	2.2045
2016	1.7475	1.5524	1.6606	1.7731	1.7203	1.6606	1.3407	1.6914	1.5524	2.0531	1.9039	2.0384
2017	1.7475	1.5916		1.9227	1.5092	1.2616	1.5524	1.5916	1.4608	1.4608	1.6274	2.1462
2018	1.7975	1.1579		2.0234	1.5916	1.4055	1.2616	1.4608	1.5092	1.6274	1.6606	2.1703
2019	1.5524	1.4055	1.2616	1.8429	1.6274	1.4055	1.4608	1.5092	1.5092	2.0813	1.6606	1.9919
2020	1.7475	1.4608				1.5524	1.3407	1.4608	1.2616	1.4055	1.9039	2.0813
2021	1.6914	1.3407	1.1579	1.7975	1.5092		1.0000	1.4608	1.3407	1.5524	1.5524	2.0531
2022	1.5092	1.2616		1.5524	1.3407		1.2616	1.1579	1.0000	1.4055	1.1579	1.7731
2023	1.5916	1.3407		1.6606	1.2616	1.0000	1.1579	1.7203	1.3407	1.5092	1.3407	1.6606
2024	1.6914	1.0000		1.8429	1.1579	1.5092	1.0000	1.3407	1.0000	1.1579	1.4055	1.9039

Citation: Annastecia C Egwuekwe*, Kelechukwu CN Dozie, Stephen O Ihekun, Glory C Nwagwu and Celestine A Igbo, Evaluating Model Performance: A Comparative Study in the Presence of Missing Data. Annal Biostat & Biomed Appli. 7(2): 2026. ABBA.MS.ID.000659.
DOI: 10.33552/ABBA.2026.07.000659.

Appendix C: Actual Data Without Missing Observations.

	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
2008	6	2	16	12	17	4	1	15	12	30	26	24
2009	6	5	7	23	20	7	4	15	11	17	14	30
2010	18	5	7	28	14	7	7	5	16	15	19	32
2011	9	7	3	17	11	9	6	7	8	14	23	39
2012	7	8	8	38	14	13	2	13	37	10	30	57
2013	15	6	8	42	20	14	5	24	6	36	37	40
2014	16	12	7	28	25	10	5	11	11	14	32	45
2015	20	12	7	30	18	5	5	13	9	18	15	42
2016	14	8	11	15	13	11	4	12	8	30	21	29
2017	14	9	6	22	7	3	8	9	6	6	10	37
2018	16	2	5	28	9	5	3	6	7	10	11	39
2019	8	5	3	18	10	5	6	7	7	32	11	26
2020	14	6	3	3	6	8	4	6	3	5	21	32
2021	12	4	2	16	7	3	1	6	4	8	8	30
2022	7	3	2	8	4	2	3	2	1	5	2	15
2023	9	4	1	11	3	1	2	13	4	7	4	11
2024	12	1	1	18	2	7	1	4	1	2	5	21

Appendix D: Transformed Data without Missing Observations.

	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
2008	1.4608	1.1579	1.7975	1.6914	1.8207	1.3407	1.0000	1.7731	1.6914	2.0531	1.9919	1.9585
2009	1.4608	1.4055	**1.4987**	1.9409	1.8844	1.5092	1.3407	1.7731	1.6606	1.8207	1.7475	2.0531
2010	1.8429	1.4055	**1.5219**	2.0234	1.7475	1.5092	1.5092	1.4055	1.7975	1.7731	1.8640	2.0813
2011	1.5916	1.5092	1.2616	1.8207	1.6606	1.5916	1.4608	1.5092	1.5524	1.7475	1.9409	2.1703
2012	1.5092	1.5524	**1.5426**	2.1584	1.7475	1.7203	1.1579	1.7203	2.1462	1.6274	2.0531	2.3516
2013	1.7731	1.4608	**1.5399**	2.2045	1.8844	1.7475	1.4055	1.9585	1.4608	2.1338	2.1462	2.1819
2014	1.7975	1.6914	1.5092	2.0234	1.9754	1.6274	1.4055	1.6606	1.6606	1.7475	2.0813	2.2369
2015	1.8844	1.6914	**1.5087**	2.0531	1.8429	1.4055	1.4055	1.7203	1.5916	1.8429	1.7731	2.2045
2016	1.7475	1.5524	1.6606	1.7731	1.7203	1.6606	1.3407	1.6914	1.5524	2.0531	1.9039	2.0384
2017	1.7475	1.5916	**1.443**	1.9227	1.5092	1.2616	1.5524	1.5916	1.4608	1.4608	1.6274	2.1462
2018	1.7975	1.1579	**1.3971**	2.0234	1.5916	1.4055	1.2616	1.4608	1.5092	1.6274	1.6606	2.1703
2019	1.5524	1.4055	1.2616	1.8429	1.6274	1.4055	1.4608	1.5092	1.5092	2.0813	1.6606	1.9919
2020	1.7475	1.4608	**1.2795**	**1.7305**	**1.4741**	1.5524	1.3407	1.4608	1.2616	1.4055	1.9039	2.0813
2021	1.6914	1.3407	1.1579	1.7975	1.5092	**1.2373**	1.0000	1.4608	1.3407	1.5524	1.5524	2.0531
2022	1.5092	1.2616	**1.1274**	1.5524	1.3407	**1.1547**	1.2616	1.1579	1.0000	1.4055	1.1579	1.7731
2023	1.5916	1.3407	**1.0383**	1.6606	1.2616	1.0000	1.1579	1.7203	1.3407	1.5092	1.3407	1.6606
2024	1.6914	1.0000	**0.9407**	1.8429	1.1579	1.5092	1.0000	1.3407	1.0000	1.1579	1.4055	1.9039