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Review Article

Impact of Missing Data on Quadratic Trend-Cycle and Seasonal Indices in Time Series Analysis: Additive Case

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Abstract

This paper discusses impact of missing data on quadratic trend cycle and seasonal indices in time series. The method adopted in this study is the Buys-Ballot procedure developed to, among other things, choice of appropriate model for decomposition of any study series, estimation of trend parameters and seasonal indices based on row, column and overall means and variances. The estimated trend parameters and seasonal indices in the presence and absence of missing data are compared. Results indicate that difference between seasonal indices in the presence and absence of missing data has significant effect. The significant differences existed on the points in which there are missing data in the column of the Buys-Ballot table. But insignificant in the trend parameters.

Keywords: Missing Data, Additive Model, Trend Parameter, Seasonal Indices, Buys-Ballot table

Introduction

A common problem that is frequently encountered in time series data is missing observations. This so since data are records taken through time. Missing data occurred because of several problems, such as technical fault or human errors (the object of observation did not give sufficient data to the observer) Pratama, et al. [1]. Brockwell and Davis [2] stated that missing data at the beginning or the end of the time series are simply ignored while intermediate missing data are considered serious flaws in the input time series. Missing observations in the time series data are very common [3]. This happens when an observation may not be made at a particular time, due to faulty equipment, cost records or a mistake, which cannot be rectified until later. When this happens, it is necessary to obtain estimate of the missing value for better understanding of the nature of the data and make possible a more accurate forecast [4]. Pratama, et al. [1], in a study on a review of missing observations handling methods on time series data observed that estimation method is probably the best option of missing data handling, since to accomplish certain work, the complete dataset is needed and some dataset have dependent variable which is impossible to delete the missing values as it can disrupt the data itself.

Cheema [5] compared different missing data handling methods (listwise deletion, mean imputation, regression imputation, maximum likelihood imputation and multiple imputation) using different methods of analysis (one sample t-test, two-sample t-test, two-way ANOVA and multiple regression). These methods, according to him are the four analytical methods that are frequently employed in educational research. However, his result did not cover handling missing values in time series data. IS Iwueze, et al. [6] recommended that the Decomposition Without Missing Value (DWMV) method be used in estimating missing values in time series analysis, this is because DWMV yielded best (in terms of the accuracy measures) estimates of the missing values when compared with both the existing methods and the two other new proposed methods (Row Mean Imputation and Column Mean Imputation). Also, DWMV combines the effects of both the trending curves and the seasonal indices unlike the other methods. This study considers the impact of missing data on Buys-Ballot estimates when trend cycle component of time series is quadratic. The Buys-Ballot is primarily used for the decomposition of a relatively short-term period such that the trend and cyclical components are jointly estimated Chatfield [7]. This estimation procedure based on the Buys-Ballot table is particularly useful since it only involves computing the column and row totals and averages of the table.

Buys-Ballot decomposition models are

Additive Model: $X_t = M_t + S_t + u_t$ (1)

Multiplicative Model: $X_t = M_t \times S_t \times u_t$ (2)

Mixed Model: $X_t = M_t \times S_t + u_t$ (3)

It is always assumed that the seasonal effect, when it exists, has a period that is, it repeats after time periods.

$S_{t+s} = S_t$, for all t (4)

For equation (1), it is convenient to make assumption that the sum of the seasonal components over a complete period is zero, ie, $\sum_{j=1}^{s} S_{t+j} = 0$ (5)

Similarly, for equations (2) and (3), the convenient variant assumption is that the sum of the seasonal components over a complete period is $s\sum_{j=1}^{s} S_{t+j} = s$ (6)

Iwueze, et al. [6] summarized the Buys-Ballot procedure called the Buys-Ballot table. A Buys-Ballot table summarizes data to show seasonal variations. Each line in the table is one period (usually a year) and each column is a season of the period/year (4 quarters, 12 months, etc). A cell (i,j), of this table contain the mean value of all observations made during the period i at the season j. To analyse the data, it is helpful to include the period and seasonal totals $(T_i \text{ and } T_j)$, period and seasonal average $\left(\bar{X}_{i.} \text{ and } \bar{X}_{.j}\right)$ period and seasonal standard deviations $(\hat{\sigma}_i \text{ and } \hat{\sigma}_j)$ as part of the Buys-Ballot table. Also included for purposes of analysis are the grand total $(T_{..})$, grand mean. $\left(\bar{X}_{..}\right)$ and pooled standard deviation $(\hat{\sigma}_{..})$ (see Table 1).

According to Wei [8], the arrangement of data in this manner in table is credited to Buys-Ballot; hence, the table has been called the Buys-Ballot table in the literature. Buys-Ballot is used to estimate the trend component and seasonal indices from the chosen descriptive time series model. According to Dozie [9], Buys Ballot procedure is computationally simple when compared with other descriptive methods. The values of the estimated trend parameters and seasonal indices are easily computed in the mixed model in time series. Iwueze and Nwogu [10] proposed the Buys-Ballot estimation procedure and for the periodic means $\left(\bar{X}_{i.}, i = 1, 2, \ldots, m\right)$ and the overall mean $\left(\bar{X}_{..}\right)$ to estimate the trend component. Seasonal means $\left(\bar{X}_{.j}, j = 1, 2, \ldots, s\right)$ and the overall mean are used to estimate the seasonal indices.

This research is restricted to time series with quadratic trend that admits the additive model using registered number of reported marriages over a period January, 2008 to December 2024. The use of Buys-Ballot table in estimation of trend parameters and seasonal indices in the presence of missing data using the methods of Iwueze and Nwogu [10] will be stated. The missing data will be estimated using the decomposition method of Iwueze, et al. [6] and the entire process of estimation will be repeated in the absence of the missing data. The impact on the Buys-Ballot estimation of quadratic trend parameters and seasonal indices in the presence and absences of the missing data will be established. The, ultimate objective of this study is to identify the impact of missing data on quadratic trend parameters and seasonal indices. The specific objectives are to: (a) estimate the missing data (b) estimate the trend parameter and seasonal indices of the data in the presence of missing observations (c) estimate the trend parameter and seasonal indices of the data in the absence of missing observations (d) compare estimates trend parameters and seasonal indices in the presence and absence of missing data. Based on the results and recommendations are made. This work contributes to the many existing solution of the problem of imputing missing data to a time series data.

Methodology

The methods adopted in this study is the Buys-Ballot procedure developed for choice of model for decomposition of any study series, estimation of trend parameters and seasonal indices and choice of appropriate transformation based on the row, column and overall means and variances, For details of Buys-Ballot procedure for time series decomposition see Wei [8], Iwueze and Nwogu [10], Nwogu, et al. [11], Dozie [9], Dozie and Uwaezuoke [12], Dozie, et al. [13], Dozie and Ijeoma [14], Dozie and Nwanya [15], Dozie and Ihekuna [16], Dozie and Ibebuogu [17], Dozie and Ibebuogu [18], Dozie and Uwaezuoke [19], Dozie and Ihekuna [20], Dozie and Ihekuna [21], Dozie [22], Dozie and Uwaezuoke [23] and Dozie [24] (Table 1).

Table 1: Buys-Ballot Table for Seasonal Time Series.

Rows(period) i	Columns(season) j								
	1	2	…	j	…	s	$T_{i.}$	$\bar{X}_{i.}$	$\hat{\sigma}_{i.}$
1	X_1	X_2	…	X_j	…	X_s	$T_{1.}$	$\bar{X}_{1.}$	$\hat{\sigma}_{1.}$
2	X_{s+1}	X_{s+2}	…	X_{s+j}	…	X_{2s}	$T_{2.}$	$\bar{X}_{2.}$	$\hat{\sigma}_{2.}$
3	X_{2s+1}	X_{2s+2}	…	X_{2s+j}	…	X_{3s}	$T_{3.}$	$\bar{X}_{3.}$	$\hat{\sigma}_{3.}$
i	$X_{(i-1)s+1}$	$X_{(i-1)s+2}$	…	$X_{(i-1)s+j}$	…	$X_{(i-1)s+s}$	$T_{i.}$	$\bar{X}_{i.}$	$\hat{\sigma}_{i.}$
m	$X_{(m-1)s+1}$	$X_{(m-1)s+2}$	…	$X_{(m-1)s+j}$	…	X_{m}	$T_{m.}$	$\bar{X}_{m.}$	$\hat{\sigma}_{m.}$
$T_{.j}$	$T_{.1}$	$T_{.2}$	…	$T_{.j}$	…	$T_{.s}$	$T_{..}$	-	-
$\bar{X}_{.j}$	$\bar{X}_{.1}$	$\bar{X}_{.2}$	…	$\bar{X}_{.j}$	…	$\bar{X}_{.s}$	-	$\bar{X}_{..}$	-
$\hat{\sigma}_{.j}$	$\hat{\sigma}_{.1}$	$\hat{\sigma}_{.2}$	…	$\hat{\sigma}_{.j}$	…	$\hat{\sigma}_{.s}$	-	-	$\hat{\sigma}_{..}$
Where s = number of seasons, m = number of periods and n = ms = number of observations									

Estimation of Missing Observation

Decomposing Without the Missing Value (DWMV)

This is one of the estimation methods for missing observations proposed by I.S. Iwueze, et al. [6]. In this method, estimates of the trend parameters and seasonal indices obtained from the remaining observations using any of the methods of time series decomposition are substituted into the expression for the missing value.

Hence, the estimates of the missing values by this method are given by:

For Additive model $\hat{X}_{(i-1)s+j} = \hat{M}_{(i-1)s+j} + \hat{S}_{(i-1)s+j}$ (7)

For multiplicative model $\hat{X}_{(i-1)s+j} = \hat{M}_{(i-1)s+j} \times \hat{S}_{(i-1)s+j}$ (8)

The trend-cycle components of the DWMV method for the quadratic curves is

$$\hat{M}_{(i-1)s+j} = \hat{a} + \hat{b}[(i-1)s+j] + \hat{c}\left[(i-1)s+j\right]^2 \quad (9)$$

Hence, the estimates of the missing values for additive model are given by;

$$\hat{X}_{ij} = \hat{a} + \hat{b}[(i-1)s+j] + \hat{c}\left[(i-1)s+j\right]^2 + \hat{S}_j \quad (10)$$

Estimation of Trend Parameters

To access the trend of the entire series, the plot of the transformed period/row means is considered. The expression of the quadratic trend is given by

$$\bar{X}_i = a + bt + ct^2 \quad (11)$$

Iwueze and Nwogu [10] stated the estimation of the trend parameters for an additive model when trend-cycle component is quadratic as;

$$\hat{a} = a' + \left(\frac{s-1}{2}\right)\hat{b} - \left(\frac{(s-1)(2s-1)}{6}\right)\hat{c} \quad (12)$$

$$\hat{b} = \frac{b'}{s} + \hat{c}(s-1) \quad (13)$$

$$\hat{c} = \frac{c'}{s^2} \quad (14)$$

Estimation of Seasonal Indices

Iwueze and Nwogu [10] gave the estimation of the seasonal indices for an additive model when trend-cycle component is quadratic as;

$$\hat{S}_j = \bar{X}_{.j} - d_j \quad (15)$$

where

$$d_j = \hat{a} + \frac{\hat{b}}{2}(n-s) + \frac{\hat{c}(n-s)(2n-s)}{6} + \left(\hat{b} + \hat{c}(n-s)\right)j + \hat{c}\, j^2 \quad (16)$$

Hence,

$$\hat{S}_j = \bar{X}_{.j} - \left[\hat{a} + \frac{\hat{b}}{2}(n-s) + \frac{\hat{c}(n-s)(2n-s)}{6} + \left(\hat{b} + \hat{c}(n-s)\right)j + \hat{c}j^2\right] \quad (17)$$

Real Life Example

The real-life example is based on monthly time series data on number of Marriages from St. Molumba's Catholic Church Owerri in Imo State for a period of seventeen (17) years (2008-2024) shown in appendix A. The 189 observations are transformed using the power transformation ($(X_t^{1-\beta})$ and arranged in a Buys-Ballot table as monthly data (S=12) and for 17years (m=17). The trend-cycle component Mt used the quadratic: $M_t = a + bt + ct^2$ with $\hat{a} = 1.6454, \hat{b} = 0.0032 \; and \; \hat{c} = -0.00003$ for the transformed data with the missing values and $\hat{a} = 2.1234, \hat{b} = 0.0153 \; and \; \hat{c} = -0.0001$ for the transformed data without the missing values (Figure 1&2).

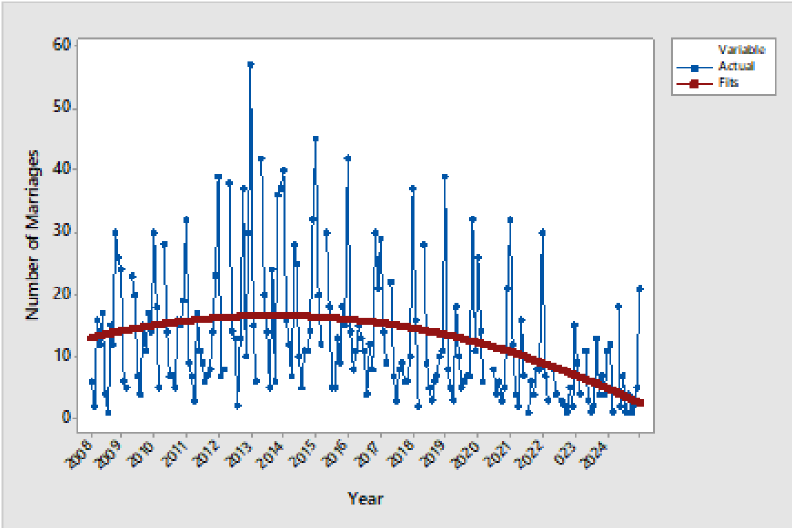

Figure 1: Plot of the Data with Missing Observations.

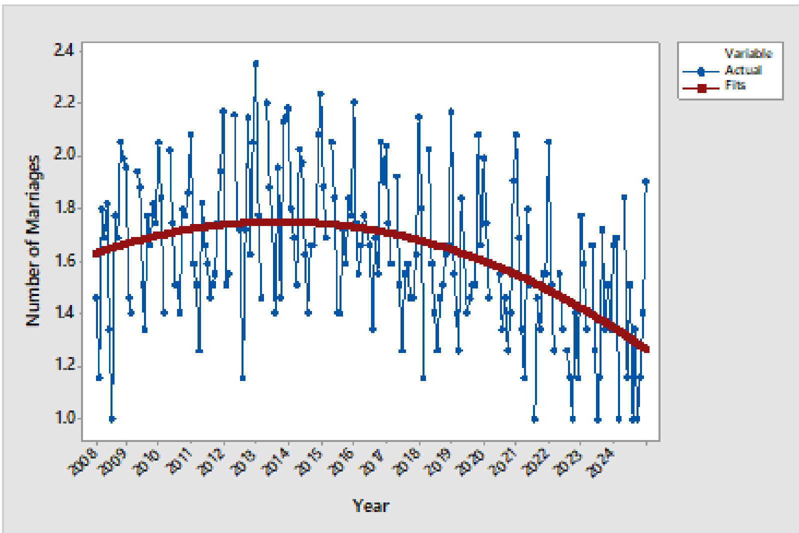

Figure 2: Plot of the Transformed Time Series Data with Missing Observations.

Estimation of Trend Parameters and Seasonal Indices of the Transformed Data with Missing Observations

Using (11)

$$\overline{X}_i = 1.6265 + 0.0424i - 0.0036i^2$$

Using (12), (13) and (14)

$$\hat{c} = \frac{-0.0036}{(12)^2} = 0.00003$$

$$\hat{b} = \frac{0.0424}{12} + (-0.00003)(12-1) = 0.00320$$

$$\hat{a} = 1.6265 + \left(\frac{12-1}{2}\right)(0.00320) - \left(\frac{(12-1)(2(12)-1)}{6}\right)(-0.00003) = 1.6454$$

Using (16) and (17)

$$d_j = 1.6454 + \frac{0.0032}{2}(189-12) + \frac{(-0.00003)(189-12)(2(189)-12)}{6} + (0.0032 + (-0.00003)(189-12))j + (-0.00003)j^2$$
$$= 1.6454 + 0.2832 - 0.3239 - 0.0021j - 0.00003j^2$$
$$d_j = 1.6047 - 0.0021j - 0.00003j^2$$

$$\hat{S}_j = \bar{X}_{.j} - \left[1.6047 - 0.0021j - 0.00003j^2\right]$$
$$= \bar{X}_{.j} - 1.6047 + 0.0021j + 0.00003j^2$$

Table 2: Seasonal Indices of the Transformed Data with the Missing Values.

j	$\bar{X}_{.j}$	$\hat{S}_{.j}$	$Adj\ \hat{S}_{.j}$
1	1.6703	0.0677	0.0364
2	1.4109	-0.1895	-0.2208
3	1.4414	-0.1567	-0.1880
4	1.8957	0.2999	0.2686
5	1.6426	0.0492	0.0179
6	1.4831	-0.1079	-0.1392
7	1.2977	-0.2908	-0.3221
8	1.5832	-0.0028	-0.0341
9	1.5021	-0.0813	-0.1126
10	1.7058	0.1251	0.0938
11	1.7536	0.1756	0.1443
12	2.0622	0.4870	0.4557
$\sum_{j=1}^{12}\hat{S}_{.j}$		0.3755	0.0000

Estimation of Missing Values of the Transformed Data

The estimates of the missing values for a quadratic trend with an additive model are given by;

$$\hat{X}_{ij} = \hat{a} + \hat{b}[(i-1)s + j] + \hat{c}\left[(i-1)s + j\right]^2 + \hat{S}_j$$

$$\hat{X}_{2,3} = 1.6454 + 0.0032[(2-1)12+3] + (-0.00003)\left[(2-1)12+3\right]^2 - 0.1880$$
$$= 1.6454 + 0.048 - 0.00675 - 0.1880 = 1.4987$$

$$\hat{X}_{3,3} = 1.6454 + 0.0032[(3-1)12+3] + (-0.00003)\left[(3-1)12+3\right]^2 - 0.1880$$
$$= 1.6454 + 0.0864 - 0.02187 - 0.1880 = 1.5219$$

$$\hat{X}_{5,3} = 1.6454 + 0.0032[(5-1)12+3] + (-0.00003)\left[(5-1)12+3\right]^2 - 0.1880$$
$$= 1.6454 + 0.1632 - 0.0780 - 0.1880 = 1.5426$$

$$\hat{X}_{6,3} = 1.6454 + 0.0032[(6-1)12+3] + (-0.00003)\left[(6-1)12+3\right]^2 - 0.1880 = 1.5399$$

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$$\hat{X}_{8,3} = 1.6454 + 0.0032[(8-1)12+3] + (-0.00003)\left[(8-1)12+3\right]^2 - 0.1880 = 1.5087$$

$$\hat{X}_{10,3} = 1.6454 + 0.0032[(10-1)12+3] + (-0.00003)\left[(10-1)12+3\right]^2 - 0.1880 = 1.4430$$

$$\hat{X}_{11,3} = 1.6454 + 0.0032[(11-1)12+3] + (-0.00003)\left[(11-1)12+3\right]^2 - 0.1880 = 1.3971$$

$$\hat{X}_{13,3} = 1.6454 + 0.0032[(13-1)12+3] + (-0.00003)\left[(13-1)12+3\right]^2 - 0.1880 = 1.2795$$

$$\hat{X}_{13,4} = 1.6454 + 0.0032[(13-1)12+4] + (-0.00003)\left[(13-1)12+4\right]^2 + 0.2686 = 1.7305$$

$$\hat{X}_{13,5} = 1.6454 + 0.0032[(13-1)12+5] + (-0.00003)\left[(13-1)12+5\right]^2 + 0.0179 = 1.4741$$

$$\hat{X}_{14,6} = 1.6454 + 0.0032[(14-1)12+6] + (-0.00003)\left[(14-1)12+6\right]^2 - 0.1392 = 1.2373$$

$$\hat{X}_{15,3} = 1.6454 + 0.0032[(15-1)12+3] + (-0.00003)\left[(15-1)12+3\right]^2 - 0.1880 = 1.1274$$

$$\hat{X}_{15,6} = 1.6454 + 0.0032[(15-1)12+6] + (-0.00003)\left[(15-1)12+6\right]^2 - 0.1392 = 1.1547$$

$$\hat{X}_{16,3} = 1.6454 + 0.0032[(16-1)12+3] + (-0.00003)\left[(16-1)12+3\right]^2 - 0.1880 = 1.0383$$

$$\hat{X}_{17,3} = 1.6454 + 0.0032[(17-1)12+3] + (-0.00003)\left[(17-1)12+3\right]^2 - 0.1880 = 0.9407$$

Since the data was transformed using power transformation, ($\left(X_t^{1-\beta}\right)$.

Where,

$$\beta = 0.7885$$

Hence we have,

$$X_t^{1-0.7885} = X_t^{0.2115}$$

Then the untransformed missing observations becomes;

$$Untransformed\ \hat{X}_t = \left(\hat{X}_t\right)^{1/0.2115}$$

$$Untransformed\ \hat{X}_{2,3} = (1.4987)^{1/0.2115} = 6.773 \approx 7$$

$$Untransformed\ \hat{X}_{3,3} = (1.5219)^{1/0.2115} = 7.2836 \approx 7$$

$$Untransformed\ \hat{X}_{5,3} = (1.5426)^{1/0.2115} = 7.7640 \approx 8$$

$$Untransformed\ \hat{X}_{6,3} = (1.5399)^{1/0.2115} = 7.700 \approx 8$$

$$Untransformed\ \hat{X}_{8,3} = (1.5087)^{1/0.2115} = 6.9897 \approx 7$$

$$Untransformed\ \hat{X}_{10,3} = (1.443)^{1/0.2115} = 5.6628 \approx 6$$

$$Untransformed\ \hat{X}_{11,3} = (1.3971)^{1/0.2115} = 4.8602 \approx 5$$

$$Untransformed\ \hat{X}_{13,3} = (1.2795)^{1/0.2115} = 3.207 \approx 3$$

$$Untransformed\ \hat{X}_{13,4} = (1.7305)^{1/0.2115} = 13.3692 \approx 13$$

$$Untransformed\ \hat{X}_{13,5} = (1.4741)^{1/0.2115} = 6.2635 \approx 6$$

$$Untransformed\ \hat{X}_{14,6} = (1.2373)^{1/0.2115} = 2.7367 \approx 3$$

$$Untransformed\ \hat{X}_{15,3} = (1.1274)^{1/0.2115} = 1.7629 \approx 2$$

$$Untransformed\ \hat{X}_{15,6} = (1.1547)^{1/0.2115} = 1.9741 \approx 2$$

$$Untransformed\ \hat{X}_{16,3} = (1.0383)^{1/0.2115} = 1.1945 \approx 1$$

$$Untransformed\ \hat{X}_{17,3} = (0.9407)^{1/0.2115} = 0.749 \approx 1$$

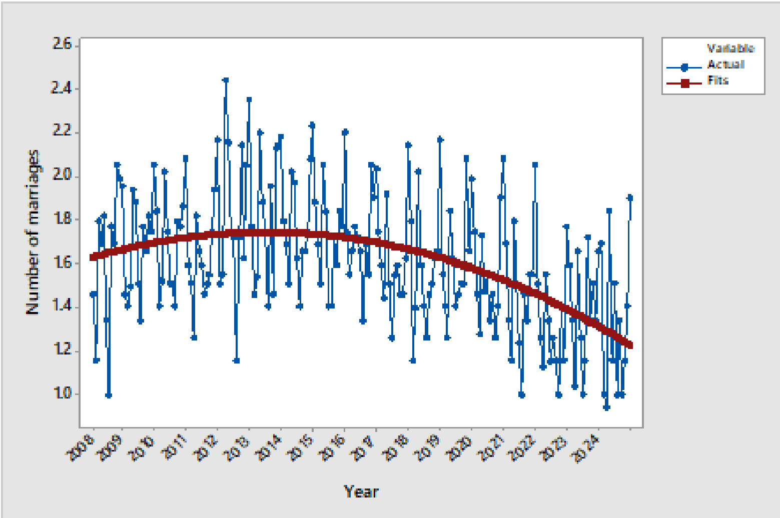

Figure 3: Plot of the Transformed Time Series Data without Missing Observations.

Estimation of Trend Parameters and Seasonal Indices of the Transformed Data Without Missing Values.

$$\bar{X}_{i\cdot} = 1.6127 + 0.0441i - 0.0038i^2$$

Using (12), (13) and (14)

$$\hat{c} = \frac{-0.0038}{(12)^2} = -0.00003$$

$$\hat{b} = \frac{0.0441}{12} + (-0.00003)(12-1) = 0.0033$$

$$\hat{a} = 1.6127 + \left(\frac{12-1}{2}\right)(0.0033) - \left(\frac{(12-1)(2(12)-1)}{6}\right)(-0.00003) = 1.6322$$

Using (16) and (17)

$$d_j = 1.6322 + \frac{0.0033}{2}(204-12) + \frac{(-0.00003)(204-12)(2(204)-12)}{6} + (0.0033 + (-0.00003)(204-12))j + (-0.00003)j^2$$
$$= 1.6322 + 0.3168 - 0.38016 - 0.00246j - 0.00003j^2$$
$$d_j = 1.56884 - 0.00246j - 0.00003j^2$$

$$\hat{S}_j = \bar{X}_{\cdot j} - \left[1.56884 - 0.00246j - 0.00003j^2\right]$$
$$= \bar{X}_{\cdot j} - 1.56884 + 0.00246j + 0.00003j^2$$

Results show that, there is no significant effect between the trend parameters with missing observations and without missing observations but there is a significant effect in the seasonal indices only at the seasons points (season three) where missing observations occurred the most in the Buys-Ballot table (Table 3&4).

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Table 3: Seasonal Indices of the Transformed Data Without Missing Values.

j	$\bar{X}_{.j}$	$\hat{S}_{.j}$	$Adj\ \hat{S}_{.j}$
1	1.6703	0.1040	0.0439
2	1.4109	-0.1529	-0.2130
3	1.3815	-0.1797	-0.2397
4	1.886	0.3275	0.2674
5	1.6327	0.0769	0.0169
6	1.4493	-0.1037	-0.1638
7	1.2977	-0.2525	-0.3125
8	1.5832	0.0360	-0.0241
9	1.5021	-0.0422	-0.1022
10	1.7058	0.1646	0.1045
11	1.7536	0.2155	0.1554
12	2.0622	0.5272	0.4672
$\sum_{j=1}^{12} \hat{S}_{.j}$		0.7206	0.0000

Table 4: Comparing the Parameters of Trend and Seasonal Indices of the Transformed Data with and without the Missing Values.

Parameters	With the missing values	Without the missing values	Differences
$\hat{a}$	1.6454	1.6322	0.0132
$\hat{b}$	0.0032	0.0033	-0.0001
$\hat{c}$	-0.00003	-0.00003	0.0000
$\hat{S}_1$	0.0364	0.0439	-0.0075
$\hat{S}_2$	-0.2208	-0.213	-0.0078
$\hat{S}_3$	-0.188	-0.2397	0.0517
$\hat{S}_4$	0.2686	0.2674	0.0012
$\hat{S}_5$	0.0179	0.0169	0.0010
$\hat{S}_6$	-0.1392	-0.1638	0.0246
$\hat{S}_7$	-0.3221	-0.3125	-0.0096
$\hat{S}_8$	-0.0341	-0.0241	-0.0100
$\hat{S}_9$	-0.1126	-0.1022	-0.0104
$\hat{S}_{10}$	0.0938	0.1045	-0.0107
$\hat{S}_{11}$	0.1443	0.1554	-0.0111
$\hat{S}_{12}$	0.4557	0.4672	-0.0115

Concluding Remarks

This work has discussed impact of missing data on quadratic trend cycle and seasonal indices in time series analysis and admits additive model. The methods adopted are Buys-Ballot Procedure and Decomposing Without Missing Value. The estimated trend parameters and seasonal indices in the presence and absence of missing data are compared. Results indicate that difference between seasonal indices in the presence and absence of missing data has significant impact. The significant differences existed on the points in which there are missing data in the column of the Buys-Ballot table. In the case of corresponding trend parameters, the presence and absence of missing data have no much differences, therefore, they are approximately the same. This shows that the missing data has no impact on trend parameter. This estimates for unobserved number of registered marriages are: (1.4987) in March 2009, (1.5219) 2010, (1.5426) 2012, (1.5399) 2013, (1.5087) 2015, (1.4430) 2017, (1.3971) 2018, (1.2795) 2020, (1.1274) 2022, (1.0383) 2023, (0.9407) 2014, (1.7305) in April 2020, (1.4741) in May 2020, (1.2373) in June 2021 and (1.1547) in June 2022.

Appendix

Appendix A: Actual Data with Missing Data.

	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
2008	6	2	16	12	17	4	1	15	12	30	26	24
2009	6	5		23	20	7	4	15	11	17	14	30
2010	18	5		28	14	7	7	5	16	15	19	32
2011	9	7	3	17	11	9	6	7	8	14	23	39
2012	7	8		38	14	13	2	13	37	10	30	57
2013	15	6		42	20	14	5	24	6	36	37	40
2014	16	12	7	28	25	10	5	11	11	14	32	45
2015	20	12		30	18	5	5	13	9	18	15	42
2016	14	8	11	15	13	11	4	12	8	30	21	29
2017	14	9		22	7	3	8	9	6	6	10	37
2018	16	2		28	9	5	3	6	7	10	11	39
2019	8	5	3	18	10	5	6	7	7	32	11	26
2020	14	6				8	4	6	3	5	21	32
2021	12	4	2	16	7		1	6	4	8	8	30
2022	7	3		8	4		3	2	1	5	2	15
2023	9	4		11	3	1	2	13	4	7	4	11
2024	12	1		18	2	7	1	4	1	2	5	21

Appendix B: Transformed Data with Missing Data.

	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
2008	1.4608	1.1579	1.7975	1.6914	1.8207	1.3407	1.0000	1.7731	1.6914	2.0531	1.9919	1.9585
2009	1.4608	1.4055		1.9409	1.8844	1.5092	1.3407	1.7731	1.6606	1.8207	1.7475	2.0531
2010	1.8429	1.4055		2.0234	1.7475	1.5092	1.5092	1.4055	1.7975	1.7731	1.8640	2.0813
2011	1.5916	1.5092	1.2616	1.8207	1.6606	1.5916	1.4608	1.5092	1.5524	1.7475	1.9409	2.1703
2012	1.5092	1.5524		2.1584	1.7475	1.7203	1.1579	1.7203	2.1462	1.6274	2.0531	2.3516
2013	1.7731	1.4608		2.2045	1.8844	1.7475	1.4055	1.9585	1.4608	2.1338	2.1462	2.1819
2014	1.7975	1.6914	1.5092	2.0234	1.9754	1.6274	1.4055	1.6606	1.6606	1.7475	2.0813	2.2369
2015	1.8844	1.6914		2.0531	1.8429	1.4055	1.4055	1.7203	1.5916	1.8429	1.7731	2.2045
2016	1.7475	1.5524	1.6606	1.7731	1.7203	1.6606	1.3407	1.6914	1.5524	2.0531	1.9039	2.0384
2017	1.7475	1.5916		1.9227	1.5092	1.2616	1.5524	1.5916	1.4608	1.4608	1.6274	2.1462
2018	1.7975	1.1579		2.0234	1.5916	1.4055	1.2616	1.4608	1.5092	1.6274	1.6606	2.1703

2019	1.5524	1.4055	1.2616	1.8429	1.6274	1.4055	1.4608	1.5092	1.5092	2.0813	1.6606	1.9919
2020	1.7475	1.4608				1.5524	1.3407	1.4608	1.2616	1.4055	1.9039	2.0813
2021	1.6914	1.3407	1.1579	1.7975	1.5092		1.0000	1.4608	1.3407	1.5524	1.5524	2.0531
2022	1.5092	1.2616		1.5524	1.3407		1.2616	1.1579	1.0000	1.4055	1.1579	1.7731
2023	1.5916	1.3407		1.6606	1.2616	1.0000	1.1579	1.7203	1.3407	1.5092	1.3407	1.6606
2024	1.6914	1.0000		1.8429	1.1579	1.5092	1.0000	1.3407	1.0000	1.1579	1.4055	1.9039

Appendix C: Actual Data without Missing Data.

	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
2008	6	2	16	12	17	4	1	15	12	30	26	24
2009	6	5	7	23	20	7	4	15	11	17	14	30
2010	18	5	7	28	14	7	7	5	16	15	19	32
2011	9	7	3	17	11	9	6	7	8	14	23	39
2012	7	8	8	38	14	13	2	13	37	10	30	57
2013	15	6	8	42	20	14	5	24	6	36	37	40
2014	16	12	7	28	25	10	5	11	11	14	32	45
2015	20	12	7	30	18	5	5	13	9	18	15	42
2016	14	8	11	15	13	11	4	12	8	30	21	29
2017	14	9	6	22	7	3	8	9	6	6	10	37
2018	16	2	5	28	9	5	3	6	7	10	11	39
2019	8	5	3	18	10	5	6	7	7	32	11	26
2020	14	6	3	3	6	8	4	6	3	5	21	32
2021	12	4	2	16	7	3	1	6	4	8	8	30
2022	7	3	2	8	4	2	3	2	1	5	2	15
2023	9	4	1	11	3	1	2	13	4	7	4	11
2024	12	1	1	18	2	7	1	4	1	2	5	21

Appendix D: Transformed Data without Missing Data.

	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
2008	1.4608	1.1579	1.7975	1.6914	1.8207	1.3407	1.0000	1.7731	1.6914	2.0531	1.9919	1.9585
2009	1.4608	1.4055	1.4987	1.9409	1.8844	1.5092	1.3407	1.7731	1.6606	1.8207	1.7475	2.0531
2010	1.8429	1.4055	1.5219	2.0234	1.7475	1.5092	1.5092	1.4055	1.7975	1.7731	1.8640	2.0813
2011	1.5916	1.5092	1.2616	1.8207	1.6606	1.5916	1.4608	1.5092	1.5524	1.7475	1.9409	2.1703
2012	1.5092	1.5524	1.5426	2.1584	1.7475	1.7203	1.1579	1.7203	2.1462	1.6274	2.0531	2.3516
2013	1.7731	1.4608	1.5399	2.2045	1.8844	1.7475	1.4055	1.9585	1.4608	2.1338	2.1462	2.1819
2014	1.7975	1.6914	1.5092	2.0234	1.9754	1.6274	1.4055	1.6606	1.6606	1.7475	2.0813	2.2369
2015	1.8844	1.6914	1.5087	2.0531	1.8429	1.4055	1.4055	1.7203	1.5916	1.8429	1.7731	2.2045
2016	1.7475	1.5524	1.6606	1.7731	1.7203	1.6606	1.3407	1.6914	1.5524	2.0531	1.9039	2.0384
2017	1.7475	1.5916	1.443	1.9227	1.5092	1.2616	1.5524	1.5916	1.4608	1.4608	1.6274	2.1462
2018	1.7975	1.1579	1.3971	2.0234	1.5916	1.4055	1.2616	1.4608	1.5092	1.6274	1.6606	2.1703
2019	1.5524	1.4055	1.2616	1.8429	1.6274	1.4055	1.4608	1.5092	1.5092	2.0813	1.6606	1.9919
2020	1.7475	1.4608	1.2795	1.7305	1.4741	1.5524	1.3407	1.4608	1.2616	1.4055	1.9039	2.0813
2021	1.6914	1.3407	1.1579	1.7975	1.5092	1.2373	1.0000	1.4608	1.3407	1.5524	1.5524	2.0531
2022	1.5092	1.2616	1.1274	1.5524	1.3407	1.1547	1.2616	1.1579	1.0000	1.4055	1.1579	1.7731
2023	1.5916	1.3407	1.0383	1.6606	1.2616	1.0000	1.1579	1.7203	1.3407	1.5092	1.3407	1.6606
2024	1.6914	1.0000	0.9407	1.8429	1.1579	1.5092	1.0000	1.3407	1.0000	1.1579	1.4055	1.9039

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