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Research Article

The Exponentiated Power Akash Distribution: Properties, Regression, and Applications to Infant Mortality Rate and COVID-19 Patients' Life Cycle

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Abstract

The new three-parameter exponentiated power Akash distribution is introduced, and some of its mathematical properties are addressed. Its parameters are estimated by maximum likelihood. A regression model is constructed based on the logarithm of the proposed distribution. The new regression model is deployed to fit COVID-19 censored data with the age of patients and diabetic index as the regressors. The usefulness of the proposed model is proved using the infant mortality rate for some selected countries in 2021.

Keywords: COVID-19 data; Exponentiated-G class; Exponentiated Power Akash distribution; Infant Mortality Rate; Model Adequacy; Regression model

Introduction

Exponentiation of power distributions has one major advantage to model data that exhibit polynomial tendencies. Distributions of lower degrees do not match data of this sort. Measures from health sciences such as the mortality rate of people living with a particular epidemic, the spread of diseases; or from engineering such as the strength of a tensile string, the life of a mechanical appliance; or from the economic sector namely income distribution of workers, inflation rate, and exchange rate of currencies of developing economies are good examples where distributions of this nature are applicable.

Relevant studies are exponentiated power Ishita by [1], exponentiated Ishita by [2], exponentiated power Lindley by [3], and exponentiated Adya by [4]. Other related studies are [5-15]. Interestingly, many of these exponentiated power transformations are on one-parameter distributions. This suggests that the Lindley class of distributions has some usefulness in modeling. Members in this class include [16-25].

Modeling some universal events such as infant mortality rate and the life cycle of COVID-19 patients is dominating the literature due to the level of concerns these events have posed to every society. Importantly, events such as these are threats to life, and hence human extinction is at the heart of modelers.

The main objective of this article is to develop a new parametric regression model that will be able to fit some skewed censored data and the rest of the article is in the following arrangement: in section 2, the new model is formulated. In section 3, some of the properties are presented. In section 4, the estimation of the uncensored data procedure is carried out. In section 5, the log-transformed regression model equivalent of the proposed distribution is derived together with the estimation. In section 6, an application to the life cycle of COVID-19 patients with a history of diabetic Mellitus with their age disparity is done. In section 7, the second application on the infant mortality rate of some countries in 2021 is also done. The paper is concluded in section 8.

Formulation of the New Model

The Power Akash (PA) distribution proposed by [26] with c.d.f and p.d.f given as follows;

$$G(x;\theta,\alpha)=1-\left[1+\frac{\theta x^{\alpha}\left(\theta x^{\alpha}+2\right)}{\theta^{2}+2}\right]e^{-\theta x^{\alpha}} \quad (1)$$

where θ, α >0. The probability density function (p.d.f) corresponding to 1 is

$$g(x;\theta,\alpha)=\frac{\alpha\theta^{3}}{(\theta^{2}+2)}(1+x^{2\alpha})x^{\alpha-1}e^{-\theta x^{\alpha}};x>0,\theta>0,\alpha>0 \quad (2)$$

The PA distribution is a two-component mixture that contains a Weilbull distribution (with shape parameter α and scale parameter θ), and a generalized gamma distribution (with shape parameters 3, α and scale parameter θ) with mixing proportion

$$p=\frac{\theta^{2}}{\theta^{2}+2}$$

The c.d.f and p.d.f of the exp-G distribution with power parameter c > 0 are given by

$$F(x;c,\xi)=G(x;\xi)^{c} \quad (3)$$

and

$$f(x;c,\xi)=cg(x;\xi)G(x;\xi)^{c-1} \quad (4)$$

respectively, where ξ is the parameter vector. By substituting equation 1 into equation 3 and equation 1 and 2 into 4, the c.d.f and p.d.f of the random variable X ~ Exponentiated Power Akash EPA (c, θ, α) are as follows:

$$F(x;c,\theta,\alpha)=\left\{1-\left[1+\frac{\theta x^{\alpha}(\theta x^{\alpha}+2)}{\theta^{2}+2}\right]e^{-\theta x^{\alpha}}\right\}^{c} \quad (5)$$

and

$$f(x;c,\theta,\alpha)=\frac{c\alpha\theta^{3}}{(\theta^{2}+2)}(1+x^{2\alpha})x^{\alpha-1}e^{-\theta x^{\alpha}}\left\{1-\left[1+\frac{\theta x^{\alpha}(\theta x^{\alpha}+2)}{\theta^{2}+2}\right]e^{-\theta x^{\alpha}}\right\}^{c-1} \quad (6)$$

respectively. The hazard rate function is given as

$$\tau(x;c,\theta,\alpha)=\frac{\frac{c\alpha\theta^{3}}{(\alpha^{2}+2)}(1+x^{2\alpha})x^{\alpha-1}e^{-\theta x^{\alpha}}\left\{1-\left[1+\frac{\theta x^{\alpha}(\theta x^{\alpha}+2)}{\theta^{2}+2}\right]e^{-\theta x^{\alpha}}\right\}^{c-1}}{1-\left\{1-\left[1+\frac{\theta x^{\alpha}(\theta x^{\alpha}+2)}{\theta^{2}+2}\right]e^{-\theta x^{\alpha}}\right\}^{c}} \quad (7)$$

Definition

(Linear Representation). Using the general binomial expansion, the pdf of the EPA distribution is given as follows;

$$f(x;c,\theta,\alpha)=\frac{c\alpha}{(\theta^{2}+2)}\sum_{j=0}^{\infty}\sum_{i=j}^{\infty}\sum_{k=0}^{j}(-1)^{i}\binom{c-1}{i}\binom{i}{j}\binom{j}{k}2^{j-k}(1+x^{2\alpha})x^{\alpha(j+k+1)-1}2^{-k}\theta^{3+j+k}e^{-(1+i)\theta x^{\alpha}} \quad (8)$$

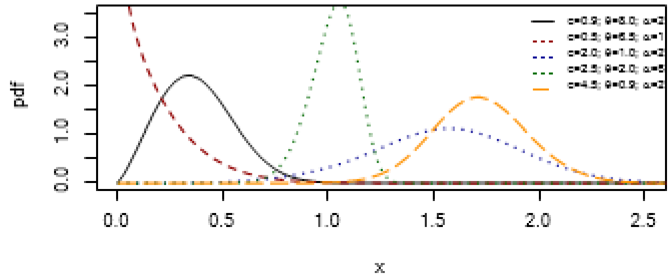

Figure 1: pdf of EPA(c, θ, α).

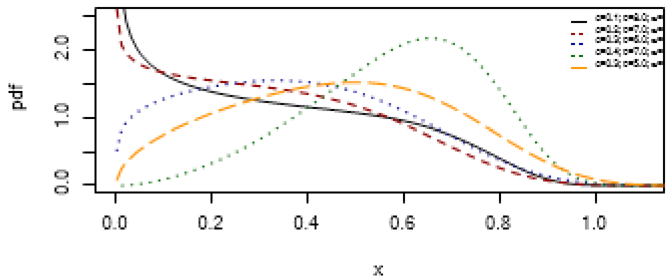

Figure 2: pdf of EPA(c, θ, α).

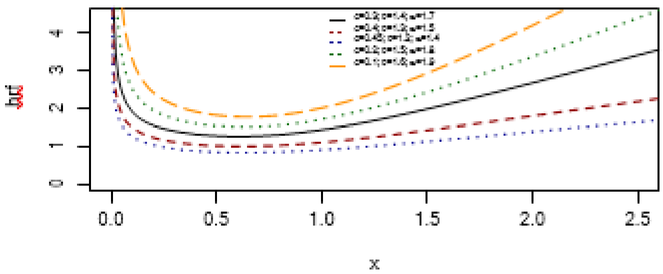

Figure 3: hazard function of EPA(c, θ, α).

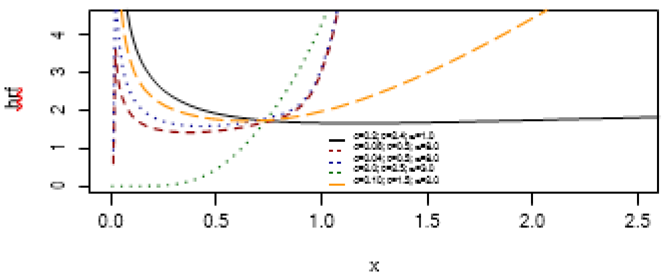

Figure 4: hazard function of EPA(c, θ, α)

The hazard function of EPA has a bathtub, increasing and decreasing shapes, see figures 3 and 4. This feature enhances the flexibility of EPA compared to the Power Akash and Akash distributions.

Properties

Definition

(Moment). The r^{th} raw moment of X ~ EPA distribution is given as $\mu_r^{'} = \int_0^{\infty} xf(x)dx$

$$\mu_r^{'} = \frac{c\alpha\theta^3}{(\theta^2+2)}\int_0^{\infty} x^r(1+x^{2\alpha})x^{\alpha-1}e^{-\theta x^{\alpha}}\left\{1-\left[1+\frac{\theta x^{\alpha}+2}{\theta^2+2}\right]e^{-\theta x^{\alpha}}\right\}^{c-1}dx \tag{9}$$

Using the change of variable technique where $x = y^{\frac{1}{\alpha}}, \mu_r^{'}$ becomes

$$\mu_r^{'} = \frac{c\alpha\theta^3}{(\theta^2+2)}\int_0^{\infty}(1+y^2)y^{(\frac{r}{\alpha}+1)-1}e^{-\theta y}\left\{1-\left[1+\frac{\theta y(\theta y+2)}{\theta^2+2}\right]e^{-\theta y}\right\}^{c-1}dy$$

Using binomial expansion on $\left\{1-\left[1+\frac{\theta y(\theta y+2)}{\theta^2+2}\right]e^{-\theta y}\right\}^{c-1}$

$$=\frac{c\alpha\theta^3}{\theta^2+2}\sum_{i=0}^{\infty}\binom{c-1}{i}(-1)^i\int_0^{\infty}\left(1+y^2\right)y^{(\frac{r}{\alpha}+1)-1}e^{1+i}\theta y\left(1+\frac{\theta y(\theta y+2)}{\theta^2+2}\right)^i dy$$

Expanding $\left(1+\frac{\theta y(\theta y+2)}{\theta^2+2}\right)^i$ using the binomial expansion

$$c=\sum_{i=0}^{\infty}\sum_{j=0}^{i}\binom{c-1}{i}\binom{i}{j}(-1)^i\frac{\theta^{j+3}}{(\theta^2+2)^{j+1}}\int_0^{\infty}(1+y^2)y^{(\frac{r}{\alpha}+j+1)-1}e^{(1+j)\theta y}(\theta+2)^j dy$$

Furtsion on $(\theta+2)^j$ we have,

$$=c\sum_{j=0}^{\infty}\sum_{i=j}^{\infty}\sum_{k=0}^{i}\binom{c-1}{i}\binom{i}{j}\binom{j}{k}(-1)^i\frac{\theta^{j+k+3}}{(\theta^2+2)^{j+1}}2^{j-k}\int_0^{\infty}\left(1+y^2\right)y^{(j+k+1)+\frac{r}{\alpha})-1}e^{-(1+i)\theta y}dy \tag{10}$$

Simplifying equation 10

$$\mu_r^{'}=\sum_{j=0}^{\infty}\sum_{i=j}^{\infty}\sum_{k=0}^{i}\binom{c-1}{i}\binom{i}{j}\binom{j}{k}(-1)^i\frac{\theta^{j+k+3}}{(\theta^2+2)^{j+1}}2^{j-k}\left\{\frac{\Gamma(j+k+1+\frac{r}{\alpha})}{((i+1)\theta)^{(j+k+1+\frac{r}{\alpha})}}-\frac{\Gamma(j+k+3+\frac{r}{\alpha})}{((i+1)\theta)^{(j+k+3+\frac{r}{\alpha})}}\right\} \tag{11}$$

The r^{th} incomplete moment of X ~ EPA defined as $m_r(v)=\int_0^{v}x^r f(x)dx$

$$m_r(v)=\frac{c\alpha\theta^3}{(\theta^2+2)}\int_0^{v}x^r(1-x^{2\alpha})x^{\alpha-1}e^{-\theta x^{\alpha}}\left\{1-\left[1+\frac{\theta x^{\alpha}+2}{\theta^2+2}\right]e^{-\theta x^{\alpha}}\right\}^{c-1}dx \tag{12}$$

Following the same steps also where $x=y^{\frac{1}{\alpha}}, m_r(v)$ becomes

$$m_r(v)=c\sum_{j=0}^{\infty}\sum_{i=j}^{\infty}\sum_{k=0}^{i}\binom{c-1}{i}\binom{i}{j}\binom{j}{k}(-1)^i\frac{\theta^{j+k+3}}{(\theta^2+2)^{j+1}}2^{j-k}\left\{\frac{\gamma(j+k+1+\frac{r}{\alpha})}{((i+1)\theta)^{(j+k+1+\frac{r}{\alpha})}}-\frac{\gamma(j+k+3+\frac{r}{\alpha})}{((i+1)\theta)^{(j+k+3+\frac{r}{\alpha})}}\right\} \quad (13)$$

Estimation

Let $x_1,x_2,........x_n$ be n independent and identically distributed random variable from the EPA distribution and $\zeta=(c,\theta,\alpha)$ vector of the unknown parameter then the log-likelihood function of ζ

$$\ell(\zeta)=\prod_{i=1}^{n}f(x,c,\theta,\alpha)$$

$$\ell(\zeta)=n\left[\log(c)+\log(\alpha)+3\log(\theta)-\log(\theta^2+2)\right]+\sum_{i=1}^{n}\log(1+x_i^{2\alpha})+(\alpha-1)\sum_{i=1}^{n}\log(x_i)-\theta\sum_{i=1}^{n}x_i^{\alpha}+(c-1)\sum_{i=1}^{n}\log\left[1-\left(1+\frac{\theta x_i^{\alpha}(\theta x_i^{\alpha}+2}{\theta^2+2}\right)e^{-\theta x_i^{\alpha}}\right] \quad (14)$$

The maximum likelihood of EPA is given as

$$\frac{\partial\ell}{\partial c}=\frac{n}{c}+\sum_{i=1}^{n}\log\left[1-\left(1+\frac{\theta x^{\alpha}(\theta x^{\alpha}+2)}{\theta^2+2}\right)e^{-\theta x^{\alpha}}\right]$$

set $\frac{\partial\ell}{\partial c}=0$

$$\hat{c}=\frac{-n}{\sum_{i=1}^{n}\log\left[1-\left(1+\frac{\theta x^{\alpha}(\theta x^{\alpha}+2)}{\theta^2+2}\right)e^{-\theta x^{\alpha}}\right]} \quad (15)$$

$$\frac{\partial\ell}{\partial\alpha}=\frac{n}{\alpha}-\sum_{i=1}^{n}\frac{2x^{2\alpha}1nx}{1-x^{2\alpha}}-\theta\sum_{i=1}^{n}x^{\alpha}1nx+(c-1)\sum_{i=1}^{n}\frac{(1+x^{2\alpha})\theta^3x^{\alpha}1nxe^{-\theta x^{\alpha}}}{(\theta^2+2)\left[1-\left(1+\frac{\theta x^{\alpha}(\theta x^{\alpha}+2)}{\theta^2+2}\right)e^{-\theta x^{\alpha}}\right]} \quad (16)$$

Equation 16 and 17 have no closed-form solutions and hence will be implemented in R using the known optim() function.

$$\frac{\partial\ell}{\partial\theta}=\frac{3n}{\theta}-\frac{2n\theta}{\theta^2+2}-\sum_{i=1}^{n}x^{\alpha}+(c-1)\sum_{i=1}^{n}\frac{\theta^2x^{\alpha}(\theta^2x^{2\alpha}+2x^{2\alpha}+2\theta x^{\alpha}+\theta^2+6)e^{-\theta x^{\alpha}}}{(\theta^2+2)^2\left[1-\left(1+\frac{\theta x^{\alpha}(\theta x^{\alpha}+2)}{\theta^2+2}\right)e^{-\theta x^{\alpha}}\right]} \quad (17)$$

Regression

Let $Y=\log(X)$ where $X\sim$ EPA (c,α,θ) defined in eq 6. Define $\alpha=\frac{1}{\sigma}$ and $\theta=e^{\frac{-\mu}{\sigma}}$, the log-Exponentiated Power Akash (LEPA) density for $y\in\mathbb{R}$ is

$$f(y;c,\sigma,\mu)=\frac{ce^{(\frac{y-3\mu}{\sigma})}\left(1+e^{\frac{2y}{\sigma}}\right)e^{-e(\frac{y-\mu}{\sigma})}}{\sigma\left(e^{-\frac{2\mu}{\sigma}}+2\right)}\left\{1-\left[1+\frac{e^{(\frac{y-\mu}{\sigma})}\left(e^{(\frac{y-\mu}{\sigma})}+2\right)}{e^{-\frac{2\mu}{\sigma}}+2}\right]e^{-e(\frac{y-\mu}{\sigma})}\right\}^{c-1} \quad (18)$$

where $c,\sigma > 0$ and $\mu \in \mathbb{R}$. If $X \sim$ EPA (c,θ,α), then $y = \log(X) \sim$ LEPA (c,σ,μ).The survival and density function of $Z = \frac{Y-\mu}{\sigma}$ are

$$s(z;c,\sigma,\mu) = 1 - \left\{1 - \left[1 + \frac{e^{z}(e^{z}+2)}{e^{-\frac{2\mu}{\sigma}}+2}\right]e^{-e^{z}}\right\}^{c} \quad (19)$$

and

$$f(z;c,\sigma,\mu) = \frac{cw(z)e^{-e^{z}}}{\sigma\left(e^{-\frac{2\mu}{\sigma}}+2\right)}\left\{1 - \left[1 + \frac{e^{z}(e^{z}+2)}{e^{-\frac{2\mu}{\sigma}}+2}\right]e^{-e^{z}}\right\}^{c-1} ;z \in \mathbb{R} \quad (20)$$

respectively, where $w(z) = e^{\frac{\sigma z+2\mu}{\sigma}}\left(1+e^{\frac{2(\sigma z+\mu)}{\sigma}}\right)$. Using equation 20,we construct a parametric regression model for the response variable Y_i and a vector of explanatory variables $V_i^{'} = (v_{i1}, v_{i2}, \ldots, v_{ip})$ as

$$y_i = V^{'}\beta + \sigma z_i,\ i = 1,2,\ldots,n \quad (21)$$

where $\mu_i = V^{'}\beta, \beta = (\beta_1,\ldots,\beta_p)^{'}$ is the vector of unknown regression coefficients and z is the random error with density in equation 12. Define the survival and density functions of $Y_i \mid V^{'}$ are

$$S(y \mid V^{'}) = 1 - \left\{1 - \left[1 + \frac{e^{z_i}}{e^{-\frac{2\mu_i}{\sigma}+1}}\right]e^{-e^{z}i}\right\}^{c} \quad (22)$$

and

$$f(y \mid V^{'}) = \frac{cw(z_i)e^{-e^{z}i}}{\sigma\left(e^{-\frac{2\mu_i}{\sigma}}+1\right)}1 - \left\{1 - \left[1 + \frac{e^{z_i}}{e^{-\frac{2\mu_i}{\sigma}+1}}\right]e^{-e^{z}i}\right\}^{c-1} \quad (23)$$

where $w(z_i) = e^{\frac{\sigma z_i - 2\mu_i}{\sigma}}\left(1+e^{\frac{2(\sigma z_i - \mu_i)}{\sigma}}\right)$ and $z_i = \frac{y_i - \mu_i}{\sigma}$

MLE of beta For Right-Censored Sample

Suppose the lifetime X of n individuals diagnosed with COVID-19 virus is EPA $(\wp)$ distributed. Let $y \mid V^{'}$ be the response variable of a parametric regression model from the EPA $(\wp)$ distribution with pdf in eq 23. To estimate the parameters β of the transformed model, the n individuals are quarantined and subjected to routine treatment at the same time. After time (t), (n – m) individuals recovers. If the lifetimes of the other m(> 0) individuals are denoted by $y_1, y_2, \ldots, y_m$ .Then, the likelihood of β can be expressed as

$$\ell(y \mid V,^{'} \beta) = \left(\prod_{i=0}^{m} f(y \mid V^{'})\right)\prod_{i=m}^{n} P(Y > t) \quad (24)$$

where $f(y|V^{'})$ is the pdf in eq 23. It is witty to write

$$P(Y>t)=P(Z>t)=\int_{t}^{\infty}\frac{cw(z_i)e^{-e^{z}i}}{\sigma\left(e^{-\frac{2\mu_i}{\sigma}}+2\right)}\left\{1-\left[1+\frac{e^{z}(e^{z}+2)}{e^{-\frac{2\mu}{\sigma}}+2}\right]e^{-e^{z}i}\right\}^{c-1}dz \quad (25)$$

where $z_i \sim$ the pdf in eq 20.

The unknown parametric regression coefficients β estimates are obtained using numerical iteration implemented in R.

Application to COVID-19 Data

The dataset comprises the lifetime (in days) of 322 individuals diagnosed with COVID-19 through RT-PCR screening in Campinas, Brazil. These data were previously studied by [1]. The response variable y_1 represents the time elapsed from the onset of symptoms until death due to COVID-19 (failure). [1] observed that about 66.45% of the observations are censored. The variables considered (f or i = 1,..., 322) include:

δ_i: censoring indicator (0 = censored, 1 = observed lifetime), v_{i1} : age (in years), and v_{i2} : diabetes mellitus (1 = yes, 0 = no or not informed). The suggested regression model for these COVID-19 data is written as

$$y_i=\beta_0+\beta_1 v_{i1}+\beta_2 v_{i2}+\sigma z_i; i=1,....,322, \quad (26)$$

The Power Prakaamy (PP) distribution by [27], exponentiated Frechet (EF) distribution by [28], power Rama (PP) distribution by [29] and power Suja (PS) distribution (new) are used to compare with the proposed exponentiated power Akash (EPA) distribution. Note that the log- of each distribution is derived following the procedure in section 5 to obtain LPP, LEF, LPR, and LPS respectively.

The result from table 1 shows that the explanatory variables age and diabetes mellitus are significant at the 5% significance level. The negative signs of β_1 and β_2 mean that older individuals or those with diabetes tend to have shorter failure times. This result is in agreement with that obtained from [1] earlier study. From table 2, the LEPA regression has the lowest criterion values hence confirming that the LEPA model provides a better fit for the COVID-19 data.

Table 1: Estimates of the Regression parameters for the COVID-19 data.

Distr	c	σ	β_0	β_1	β_2
LEPA	0.5843 (0.5845)	0.8380 (0.5255)	3.7801 (0.9222)	-0.0187 (0.0039) $<3.0602\times10^{-6}$	-0.2865 (0.1232) [<0.0206]
LPP	1	1.3760 (0.0912)	3.6182 (0.3140)	-0.0285 (0.0042) $<4.149\times10^{-11}$	-0.4074 (0.1340) [<0.0026]
LEF	154.1795 (116.3051)	3.8146 (0.4878)	10.9724 (1.3599)	-0.0212 (0.0413) $<4.8539\times10^{-7}$	-0.3015 (0.1454) [<0.0389]
LPR	1	1.3356 (0.0866)	3.5766 (0.3052)	-0.0273 (0.0040) $<6.206\times10^{-11}$	-0.4021 (0.1308) [<0.0023]
LPS	1	1.5323 (0.0848)	1.9531 (0.2880)	-0.0185 (0.0036) $<4.4779\times10^{-7}$	-0.2656 (0.1237) [<0.0325]

Table 2: Measures of model performance.

Distr	AIC	CAIC	BIC	HQIC
LEPA	433.6238	433.981	452.4966	441.1584
LPP	645.113	645.3796	660.2112	651.1407

Citation: Nwankwo M Peace, Bright Chimezie Nwankwo and Okechukwu J Obulezi*. The Exponentiated Power Akash Distribution: Properties, Regression, and Applications to Infant Mortality Rate and COVID-19 Patients' Life Cycle. Annal Biostat & Biomed Appli. 5(4): 2023. ABBA.MS.ID.000620. DOI: 10.33552/ABBA.2023.05.000620.

LEF	441.9085	442.2652	460.7812	449.4431
LPR	641.4597	641.7264	656.5579	647.4874
LPS	436.1518	436.4185	451.25	442.1795

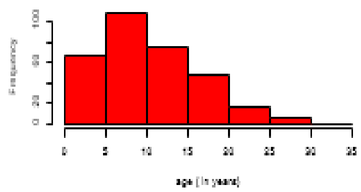

Figure 5: histogram for COVID-19 data.

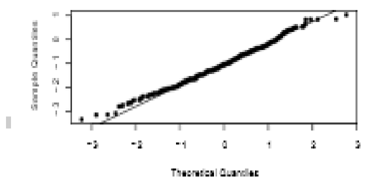

Figure 6: QQ plot for the COVID-19 data

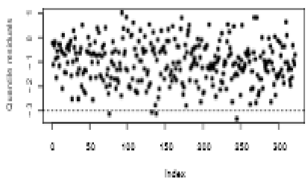

Figure 7: Quantile Residual plot for the COVID-19 data.

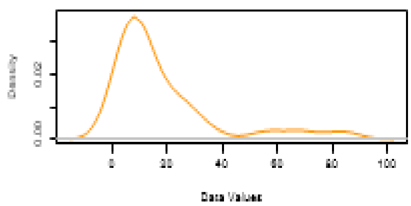

Figure 8: kernel density for infant mor-tality data.

Application to Infant Mortality Rate Data

The data on infant mortality rate per 1,000 live births for a few chosen nations in 2021, as reported by https://data.worldbank.org/indicator/SP.DYN.IMRT.IN This real data set is presented as

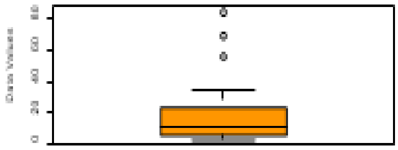

Figure 9: Boxplot for infant mortality data.

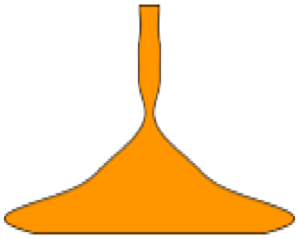

Figure 10: Violin plot for infant mortality data

A quick observation of the non-parametric plots in figures 8, 9, and 10 reveals that the infant mortality rate is right-skewed data. Probabil- ities taper off more slowly for higher values.

Table 3: Model Adequacy and Fitness Measures for the Infant Mortality Rate Data.

Distr	NLL	AIC	CAIC	BIC	HQIC	W^*	A^*	K-S	P-value	Rank
EPA	102.58	211.016	212.059	214.913	212.172	0.038	0.261	0.096	0.9658	1
PP	106.07	216.132	216.632	218.724	216.902	0.112	0.731	0.164	0.4596	5
EF	102.59	211.174	212.217	215.061	212.330	0.039	0.269	0.098	0.9583	2
PR	105.80	215.602	216.102	218.193	216.372	0.113	0.741	0.160	0.4933	3
PS	105.9	215.796	216.296	218.388	216.567	0.112	0.732	0.162	0.4797	4

Using the following information criteria; Akaike information criterion (BIC), Corrected Akaike Information criterion (CAIC), Bayesian Infor- mation Criterion (BIC), Hannan–Quinn information criterion (HQIC), the adequacy of the model was proved since the proposed distribution has minimum value for each of the criteria. The K-S, Cramer von misses W^*, Anderson Darling statistics A^*, and p-value for the proposed distribution show evidence that the new distribution fits the given data more than the competitors.

Table 4: MLEs of the unknown parameters using Infant Mortality Data.

Distr	c	θ	α
EPA	7823.7103 (14195.41)	7.4998 (1.7445)	0.1412 (0.0335)
PP	1	1.4559	0.5054

EF	1.7433 (2.3948)	(0.1947) 12.1775 (15.1142)	(0.0463) 0.9572 (0.6840)
PR	1	0.7371 (0.1383)	0.5964 (0.0615)
PS	1	1.0853 (0.1698)	0.5429 (0.0527)

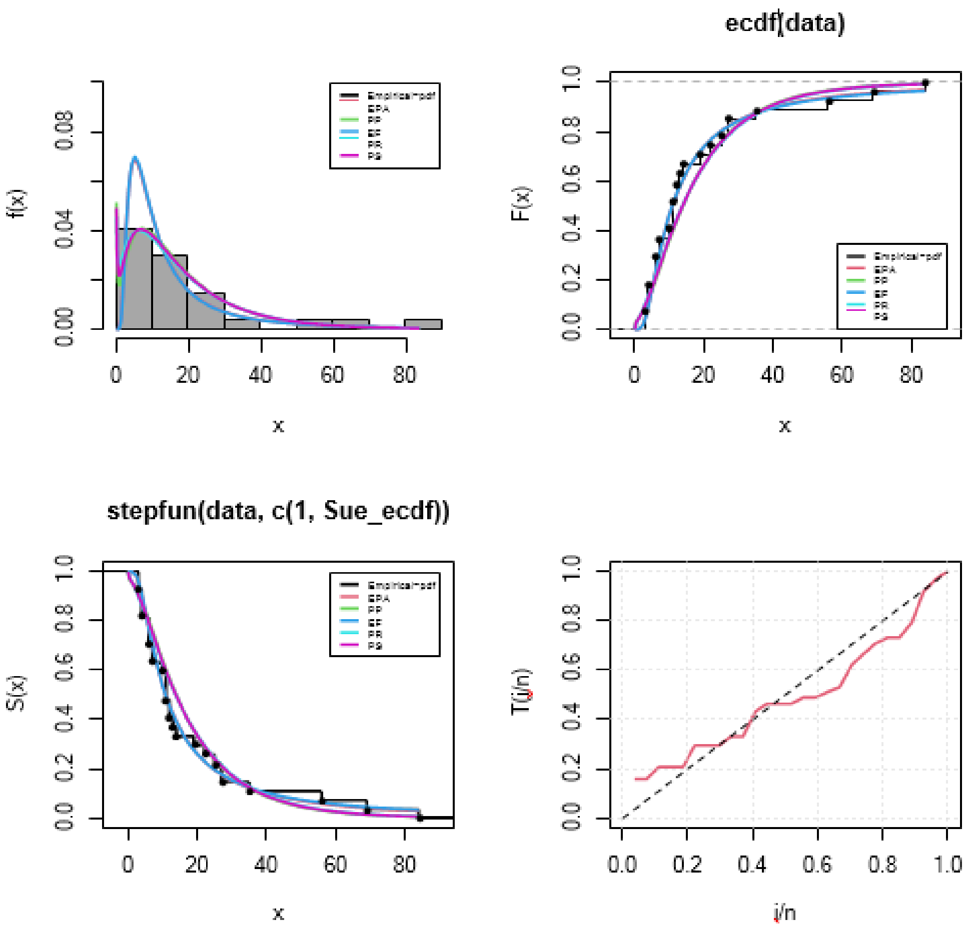

Figure 11: density, cdf, survival function, and TTT plots of the infant mortality rate data.

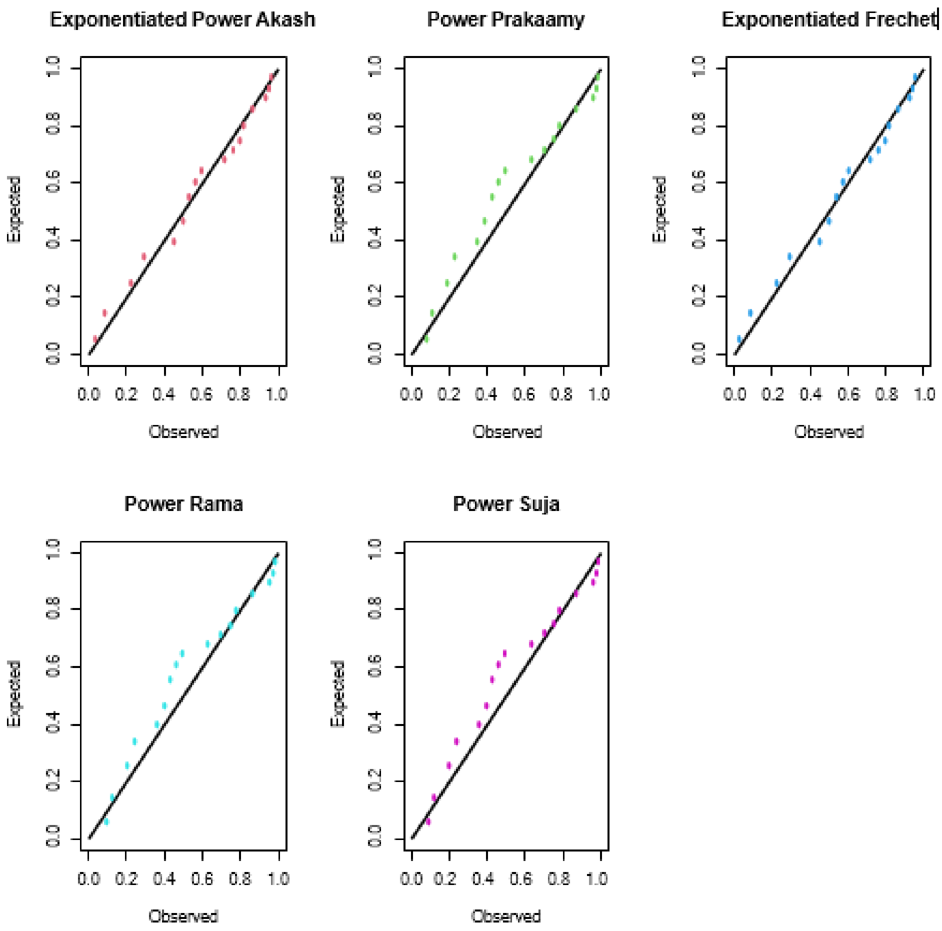

Figure 12: pp plots of the infant mortality rate data.

Conclusion

In this article, a new lifetime distribution with the potential of modeling data with inherent polynomial nature as well as skewed data. The properties of the proposed distribution were derived and the log-transformation of the proposed distribution to develop a parametric regression model was carried out. The maximum likelihood estimation aided the estimation process for uncensored samples while the procedure for the estimation of the unknown parameters when data is censored was also shown. Essentially, the censored COVID-19 data set with the age of patients and diabetic mellitus index was deployed to justify the importance of the distribution. Furthermore, the distribution was fitted to the data on infant mortality rate (below age 5 years) reported for some countries by the World Health Organization in 2021. The distribution performs pretty well in both instances of application.

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Conflict of Interest

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