ISSN: 2641-6336
DOI: 10.33552/ABBA.2020.04.000591
Annals of
Biostatistics & Biometric Applications
Iris Publishers

Review Article

General Fifth M-Zagreb Indices and General Fifth M-Zagreb Polynomials of Dyck-56 Network

Fozia Bashir Farooq*

Department of Mathematics, Al-Imam Mohammad Ibn Saud Islamic University, Riyadh, Saudi Arabia

***Corresponding author:** Fozia Bashir Farooq, Department of Mathematics, Al-Imam Mohammad Ibn Saud Islamic University, Riyadh, Saudi Arabia.

Received Date: August 31, 2021
Published Date: September 23, 2021

Abstract

A topological index is a type of molecular index that is calculated from the molecular graph of chemical structure. A topological index relates chemical structure with its underlying physical, biological and chemical properties. This article deals with General fifth M1 and General fifth M2–Zagreb indices and General fifth M1 and general fifth M2–Zagreb polynomials of Dyck-56$n\times n$ network.

Keywords: Dyck-56 Network; Zagreb index; Zagreb polynomial

Introduction

A chemical graph can be represented by polynomial, a numerical value or a matrix form. Topological indices are major source of the relationship between physical and biological properties and behavior of chemical structures. Topological indices are used in development quantitative structure activity relationships in which properties of molecules are correlated with their chemical structure. Let $G(V, E)$ be a graph with vertex set V and edge set E, with $|V| = m$ and $|E| = n$. Let d_u represent the degree of a vertex u in graph G. S_u stands for sum of degrees of the vertices incident with vertex u in G. Milan Randic [1] introduced the first degree-based Topological index, i.e Randic Index, Defined as;

$$X(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{d_u d_v}} \tag{1}$$

Furthermore, Bollabas and Erdos [2] introduced general Randic index as;

$$R_\alpha(G) = \sum\nolimits_{uv\in E(G)} \left(d_u d_v\right)^\alpha \tag{2}$$

Gutman and Trinajstic [3] introduced first and second Zagreb indices as;

$$M_1(G) = \sum_{uv\in E(G)} [d_u + d_v] \tag{3}$$

$$M_2(G) = \sum_{uv\in E(G)} [d_u d_v] \tag{4}$$

Recently in [4], Kulli introduced new indices that are general fifth M_1- Zagreb indices denoted by $M_1{}^\alpha G_5(G)$ and general fifth M_2-Zagreb indices denoted by $M_2{}^\alpha G_5(G)$ defined as;

$$M_1^\alpha G_5(G) = \sum_{uv\in E(G)} [s_u(G) + s_v(G)]^\alpha \tag{5}$$

$$M_2^\alpha G_5(G) = \sum_{uv\in E(G)} [s_u(G) s_v(G)]^\alpha \tag{6}$$

The general fifth M_1- Zagreb polynomial denoted by $M_1{}^\alpha G_5(G, x)$ and general fifth M_2- Zagreb polynomial denoted by $M^\alpha{}_2 G_5(G, x)$ is defined as;

$$M_1^{\alpha}G_5(G,x)=\sum_{uv\in E(G)} x^{[s_u(G)+s_v(G)]^{\alpha}} \quad (7)$$

$$M_2^{\alpha}G_5(G,x)=\sum_{uv\in E(G)} x^{[s_u(G)s_v(G)]^{\alpha}} \quad (8)$$

Dyck-56$_{n\times n}$ Networks were built up by unit representation of Dyck–56 (Dyck 1880) by identification procedure. Further detail about formation of Dyck Network can be found in [5]. Dyck graph is the 3-regular graph with 32 vertices and 48 edges, having chromatic number 2 and chromatic index 3 with diameter 5. Further details of formation of Dyck- 56$n\times n$ network can be found in [5,6]. Selvan and Naranyakar in [7] calculated general Randic Index, First Zagreb index, ABC and GA indices for Dyck-x, ABC and GA indices for Dyck-56$_{n\times n}$ network.

Main Results

Figure 1 Dyck-56$_{n\times n}(A)$ Network has $12n^2+4n$ vertices and $18n^2-2n$ edges. Based on the sum of the degrees of incident vertices to the end vertices of each edge, there are seven types of edges of Dyck-56$_{n\times n}(A)$ Network. This edge partition is presented in (Table 1) [8-10].

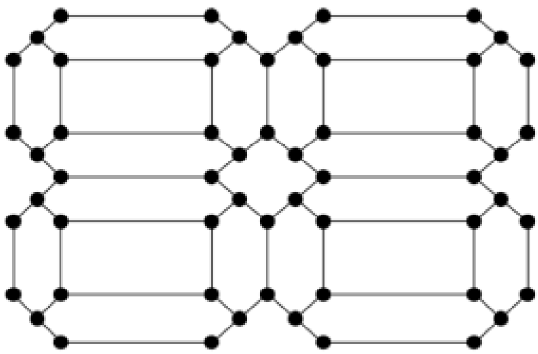

Figure 1: Dyck-562×2(A) Network.

Table1: Edge Partition of Dyck-56$_{n\times n}(A)$ Network.

(S_u, S_v)	Frequency
(5,5)	$4n$
(5,7)	8
(5,8)	$8(n-1)$
(7,9)	4
(8,9)	$16(n-1)$
(9,9)	$18n^2-26n+12$

Theorem 1: Let G be the graph Dyck-56$_{n\times n}(A)$ Network, and the general fifth M_1- Zagreb index for G is

$$M_1^{\alpha}G_5(G)=2.2^{\alpha}[2n.5^{\alpha}+4.6^{\alpha}+2.8^{\alpha}+(9n^2-13n+6)9^{\alpha}+4(n-1)(2.13^{\alpha}+4.17^{\alpha})]$$

Proof. Let G: = Dyck-56$_{n\times n}(A)$ Network; the above result can be found by using Table 1 and equation 5.

So,

$$M_1^{\alpha}G_5(G)=\sum_{uv\in E(G)}[s_u(G)+s_v(G)]^{\alpha}$$

$$=4n[5+5]^{\alpha}+8[5+7]^{\alpha}+8(n-1)[5+8]^{\alpha}+4[7+9]^{\alpha}+16(n-1)[8+9]^{\alpha}+(18n^2-26n+12)[9+9]^{\alpha}$$

$$=4n[10]^{\alpha}+8[12]^{\alpha}+8(n-1)[13]^{\alpha}+4[16]^{\alpha}+16(n-1)[17]^{\alpha}+(18n^2-26n+12)[18]^{\alpha}$$

Which further reduces to

$$M_1^{\alpha}G_5(G)=2.2^{\alpha}[2n.5^{\alpha}+4.6^{\alpha}+2.8^{\alpha}+(9n^2-13n+6)9^{\alpha}+4(n-1)(2.13^{\alpha}+4.17^{\alpha})]$$

Theorem 2: Let G be the graph Dyck-56$_{n\times n}(A)$ Network, and the general fifth M_2- Zagreb index for G is

$$M_2^{\alpha}G_5(G)=4[n.5^{\alpha}+2.7^{\alpha}+2(n-1)8^{\alpha}]5^{\alpha}+2[2.7^{\alpha}+8(n-1)8^{\alpha}+(9n^2-13n+6)9^{\alpha}]9^{\alpha}$$

Proof. Let G be Dyck-56$_{n\times n}(A)$ Network. The above result can be found using (Table 1) and equation 6.

So,

$$M_2^{\alpha}G_5(G)=\sum_{uv\in E(G)}[s_u(G).s_v(G)]^{\alpha}$$

$$=4n[5\times5]^{\alpha}+8[5\times7]^{\alpha}+8(n-1)[5\times8]^{\alpha}+4[7\times9]^{\alpha}+16(n-1)[8\times9]^{\alpha}+(18n^2-26n+12)[9\times9]^{\alpha}$$

$$= 4n[25]^{\alpha} + 8[35]^{\alpha} + 8(n-1)[40]^{\alpha} + 4[63]^{\alpha} + 16(n-1)[72]^{\alpha} + (18n^2 - 26n + 12)[81]^{\alpha}$$

Which further reduces to

$$M_2^{\alpha}G_5(G) = 4[n.5^{\alpha} + 2.7^{\alpha} + 2(n-1)8^{\alpha}]5^{\alpha} + 2[2.7^{\alpha} + 8(n-1)8^{\alpha} + (9n^2 - 13n + 6)9^{\alpha}]9^{\alpha}$$

Theorem 3: Let G be the graph Dyck-56$_{n\times n}$(A) Network, and the general fifth M_1- Zagreb polynomial for G is $M_1^{\alpha}G_5(G,x) = 4[nx^{[10]^{\alpha}} + 2x^{[12]^{\alpha}} + x^{[16]^{\alpha}}] + (18n^2 - 26n + 12)x^{[18]^{\alpha}} + [8(n-1)\{x^{[13]^{\alpha}} + 2x^{[17]^{\alpha}}\}]$

Proof. Let G be Dyck-56$_{n\times n}$(A) Network. The above result can be found using (Table 1) and equation 7.

So,

$$M_1^{\alpha}G_5(G,x) = \sum_{uv\in E(G)} x^{[s_u(G)+s_v(G)]^{\alpha}}$$

$$= 4nx^{[5+5]^{\alpha}} + 8x^{[5+7]^{\alpha}} + 8(n-1)x^{[5+8]^{\alpha}} + 4x^{[7+9]^{\alpha}} + 16(n-1)x^{[8+9]^{\alpha}} + (18n^2 - 26n + 12)x^{[9+9]^{\alpha}}$$

$$= 4nx^{[10]^{\alpha}} + 8x^{[12]^{\alpha}} + 8(n-1)x^{[13]^{\alpha}} + 4x^{[16]^{\alpha}} + 16(n-1)x^{[17]^{\alpha}} + (18n^2 - 26n + 12)x^{18]^{\alpha}}$$

Which after simplification is

$$M_1^{\alpha}G_5(G,x) = 4[nx^{[10]^{\alpha}} + 2x^{[12]^{\alpha}} + x^{[16]^{\alpha}}] + (18n^2 - 26n + 12)x^{[18]^{\alpha}} + [8(n-1)\{x^{[13]^{\alpha}} + 2x^{[17]^{\alpha}}\}]$$

Theorem 4: Let G be the graph Dyck-56$_{n\times n}$(A) Network, The general fifth M_2- Zagreb polynomial for G is

$$M_2^{\alpha}G_5(G,x) = 4[nx^{[25]^{\alpha}} + x^{[63]^{\alpha}}] + (18n^2 - 26n + 12)x^{[81]^{\alpha}} + 8(n-1)[x^{[40]^{\alpha}} + 2x^{[72]^{\alpha}}]$$

Proof. Let G be Dyck-56$n\times n$(A) Network. The above result can be found by using (Table 1) and equation 7.

So,

$$M_2^{\alpha}G_5(G,x) = \sum_{uv\in E(G)} x^{[s_u(G)\times s_v(G)]^{\alpha}}$$

$$= 4nx^{[5\times5]^{\alpha}} + 8x^{[5\times7]^{\alpha}} + 8(n-1)x^{[5\times8]^{\alpha}} + 4x^{[7\times9]^{\alpha}} + 16(n-1)x^{[8\times9]^{\alpha}} + (18n^2 - 26n + 12)x^{[9\times9]^{\alpha}}$$

$$= 4nx^{[25]^{\alpha}} + 8x^{[35]^{\alpha}} + 8(n-1)x^{[40]^{\alpha}} + 4x^{[63]^{\alpha}} + 16(n-1)x^{[72]^{\alpha}} + (18n^2 - 26n + 12)x^{[81]^{\alpha}}$$

This after simplification is

$$M_2^{\alpha}G_5(G,x) = 4[nx^{[25]^{\alpha}} + x^{[63]^{\alpha}}] + (18n^2 - 26n + 12)x^{[81]^{\alpha}} + 8(n-1)[x^{[40]^{\alpha}} + 2x^{[72]^{\alpha}}]$$

Figure 2 Dyck-56$_{n\times n}$(B) Network has $18n^2 - 3n$ vertices and $24n^2 - 4n + 8$ edges. On the base of sum of degrees of incident vertices to end vertices of each edge there are seven types of edges of Dyck-56$_{n\times n}$(B) Network. This edge partition is given in (Table 2) [11-13].

Theorem 5: Let G be the graph Dyck-56$_{n\times n}$(B) Network, The general fifth M_1- Zegreb index for G is $M_1^{\alpha}G_5(G) = 8(n-1)[3]^{\alpha}[5^{\alpha} + 7^{\alpha}] + [2]^{\alpha+2}[2\times8^{\alpha} + 2n^2 9^{\alpha} + (n^2 - n)10^{\alpha}] + (4n+8)[13]^{\alpha} + (12n^2 - 20n + 8)[23]^{\alpha}$

Proof. Let G be Dyck-56$n\times n$(A) Network, The above result can be found by using (Table 1) and equation 5.

So,

$$M_1^{\alpha}G_5(G) = \sum_{uv\in E(G)} [s_u(G) + s_v(G)]^{\alpha}$$

$$= (4n+8)[6+7]^{\alpha} + 8(n-1)[6+9]^{\alpha} + 8[7+9]^{\alpha} + 8n^2[9+9]^{\alpha} + 4n(n-1)[9+11]^{\alpha} + 8(n-1)[9+12]^{\alpha} + (12n^2 - 20n + 8)[11+12]^{\alpha}$$

$$= (4n+8)[13]^{\alpha} + 8(n-1)[15]^{\alpha} + 8[16]^{\alpha} + 8n^2[18]^{\alpha} + 4n(n-1)[20]^{\alpha} + 8(n-1)[21]^{\alpha} + (12n^2 - 20n + 8)[23]^{\alpha}$$

Which further reduces to

$$M_1^{\alpha}G_5(G) = 8(n-1)[3]^{\alpha}[5^{\alpha} + 7^{\alpha}] + [2]^{\alpha+2}[2\times8^{\alpha} + 2n^2 9^{\alpha} + (n^2 - n)10^{\alpha}] + (4n+8)[13]^{\alpha} + (12n^2 - 20n + 8)[23]^{\alpha}$$

Theorem 6: Let G be the graph Dyck-56$_{n\times n}$(B) Network. The general fifth M_2- Zegreb index for G is

$$M_2^{\alpha}G_5(G) = 8(n-1)54^{\alpha}[1+2]^{\alpha} + 4\times9^{\alpha}[2\times7^{\alpha} + 2n^2 9^{\alpha} + (n^2 - n)11^{\alpha}] + (4n+8)[42]^{\alpha} + (12n^2 - 20n + 8)[132]^{\alpha}$$

Proof. Let G denotes Dyck-56$_{n\times n}$(B) Network. The above result can be found by using (Table 2) and equation 6.

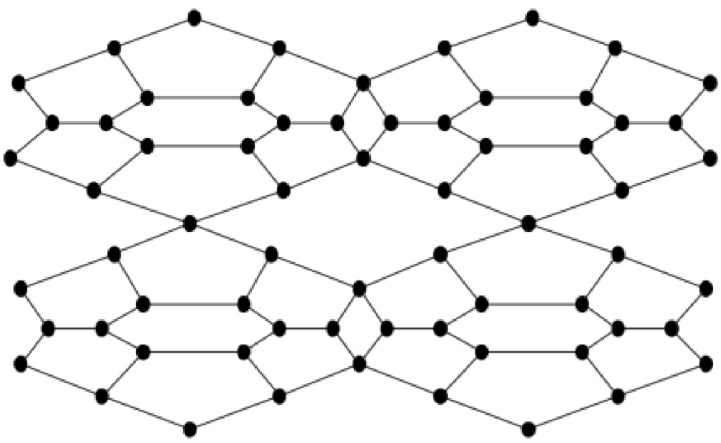

Figure2: Dyck-562×2(A) Network

Table2: Edge Partition of Dyck-56$_{n\times n}$(B) Network.

(S_u, S_v)	Frequency
(6,7)	$4n + 8$
(6,9)	$8(n - 1)$
(7,9)	8
(9,9)	$8n^2$
(9,11)	$4n\ (n - 1)$
(9,12)	$8(n - 1)$
(11,12)	$12n^2 - 20n + 8$

So,

$$M_1^{\alpha}G_5(G) = \sum_{uv\in E(G)} [s_u(G).s_v(G)]^{\alpha}$$

$$= (4n+8)[6\times 7]^{\alpha} + 8(n-1)[6\times 9]^{\alpha} + 8[7\times 9]^{\alpha} + 8n^2[9\times 9]^{\alpha} + (4n^2-4n)[9\times 11]^{\alpha} + 8(n-1)[9\times 12]^{\alpha} + (12n^2-20n+8)[11\times 12]^{\alpha}$$

$$= (4n+8)[42]^{\alpha} + 8(n-1)[54]^{\alpha} + 8[63]^{\alpha} + 8n^2[81]^{\alpha} + (4n^2-4n)[99]^{\alpha} + 8(n-1)[108]^{\alpha} + (12n^2-20n+8)[132]^{\alpha}$$

Which further reduces to

$$M_2^{\alpha}G_5(G) = 8(n-1)54^{\alpha}[1+2]^{\alpha} + 4\times 9^{\alpha}[2\times 7^{\alpha} + 2n^2 9^{\alpha} + (n^2-n)11^{\alpha}] + (4n+8)[42]^{\alpha} + (12n^2-20n+8)[132]^{\alpha}$$

Theorem 7: Let G be the graph Dyck-56$_{n\times n}$(B) Network, The general fifth M_1- Zagreb polynomial for G is

$$M_1^{\alpha}G_5(G,x) = (4n+8)x^{[13]^{\alpha}} + 8(n-1)[x^{[15]^{\alpha}} + x^{[21]^{\alpha}}] + 4[2x^{[16]^{\alpha}} + 2n^2.x^{[18]^{\alpha}} + n(n-1)x^{[20]^{\alpha}} + (18n^2-26n+12)x^{[23]^{\alpha}}$$

Proof. Let G is Dyck-56$_{n\times n}$(B) Network, The above result can be found by using (Table 2) and equation 7.

So,

$$M_1^{\alpha}G_5(G,x) = \sum_{uv\in E(G)} x^{[s_u(G)+s_v(G)]^{\alpha}}$$

$$= (4n+8)x^{[6+7]^{\alpha}} + 8(n-1)x^{[6+9]^{\alpha}} + 8x^{[7+9]^{\alpha}} + 8n^2x^{[9+9]^{\alpha}} + 4n(n-1)x^{[9+11]^{\alpha}} + 8(n-1)x^{[9+12]^{\alpha}} + (18n^2-26n+12)x^{[11+12]^{\alpha}}$$

$$= (4n+8)x^{[13]^{\alpha}} + 8(n-1)x^{[15]^{\alpha}} + 8x^{[16]^{\alpha}} + 8n^2x^{[18]^{\alpha}} + 4n(n-1)x^{[20]^{\alpha}} + 8(n-1)x^{[21]^{\alpha}} + (18n^2-26n+12)x^{[23]^{\alpha}}$$

Which after simplification is

$$M_1^{\alpha}G_5(G,x) = (4n+8)x^{[13]^{\alpha}} + 8(n-1)[x^{[15]^{\alpha}} + x^{[21]^{\alpha}}] + 4[2x^{[16]^{\alpha}} + 2n^2.x^{[18]^{\alpha}} + n(n-1)x^{[20]^{\alpha}} + (18n^2-26n+12)x^{[23]^{\alpha}}$$

Theorem 8: Let G be the graph Dyck-56$_{n\times n}$(B) Network, The general fifth M_2- Zegreb polynomial for G is

$$M_1^{\alpha}G_5(G,x) = (4n+8)x^{[42]^{\alpha}} + 8(n-1)[x^{[54]^{\alpha}} + x^{[108]^{\alpha}}] + 4[x^{[63]^{\alpha}} + 2n^2x^{[81]^{\alpha}} + (4n+8)x^{[99]^{\alpha}}] + (18n^2-26n+12)x^{[132]^{\alpha}}$$

Proof. Let G be Dyck-56$_{n\times n}$(B) Network. The above result can be found by using (Table 2) and equation 8.

So,

$$M_1^{\alpha}G_5(G,x) = \sum_{uv\in E(G)} x^{[s_u(G).s_v(G)]^{\alpha}}$$

$$= (4n+8)x^{[6\times 7]^{\alpha}} + 8(n-1)x^{[6\times 9]^{\alpha}} + 8x^{[7\times 9]^{\alpha}} + 8n^2x^{[9\times 9]^{\alpha}} + (4n^2-4n)x^{[9\times 11]^{\alpha}} + 8(n-1)x^{[9\times 12]^{\alpha}} + (18n^2-26n+12)x^{[11\times 12]^{\alpha}}$$

$$= (4n+8)x^{[42]^{\alpha}} + 8(n-1)x^{[54]^{\alpha}} + 8x^{[63]^{\alpha}} + 8n^2x^{[81]^{\alpha}} + (4n^2-4n)x^{[99]^{\alpha}} + 8(n-1)x^{[108]^{\alpha}} + (18n^2-26n+12)x^{[132]^{\alpha}}$$

Which after simplification is

$$M_1^{\alpha}G_5(G,x) = (4n+8)x^{[42]^{\alpha}} + 8(n-1)[x^{[54]^{\alpha}} + x^{[108]^{\alpha}}] + 4[x^{[63]^{\alpha}} + 2n^2x^{[81]^{\alpha}} + (4n+8)x^{[99]^{\alpha}}] + (18n^2-26n+12)x^{[132]^{\alpha}}$$

Conclusion

Degree based topological indices like General fifth M_1 and General fifth M_2 Zagreb indices for Dyck-56 Network are found in this paper. These indices are useful in study of QSAR/QSPR. Furthermore, General fifth M_1 and General fifth M_2-Zagreb polynomials for Dyck-56 Networks are found. These indices and polynomials are useful for the study to understand correlation between physical structures with chemical properties.

Acknowledgement

None.

Conflict of Interest

No conflict of interest.

References

1. Randic M (1975) On Characterization of molecular branching. J Am Chem Soc 97: 6609–6615.
2. A Graovac, M Ghorbani, MA Hosseinzadeh (2011) Computing _fth geometric- arithmetic index of nanostar dendrimers. Journal of Mathematical Nanoscience1(1): 33-42.
3. Bollobás, B, Erdös P (1998) Graphs of extermal weights. Ars Comb 50: 225-233.
4. Gutman I, Trinajstic N (1972) Graph theory and molecular orbitals. Total φ-electron energy of alternant hydrocarbons. Chem Phys Lett 17(4): 535-538.
5. MV Diudea, Cs L Nagy (2013) Diamond and related Nanostructures, Springer.
6. Mircea V Diudea, Katalin Nagy, Monica L Pop, F Gholami Nezhaad, AR Ashrafi (2010) Omega and PIv Polynomial in Dyck Graph-like Z(8)-Unit Networks. Int J Nanosci Nanotechnol 6(2): 97-103.
7. D Selwan, KP Prayankar (2020) On Topological Properties of Dyck-56 Networks. International Journal of Applied Graph Theory 4(1): 1-15.
8. VR Kulli (2017) General Fifth M-Zagreb Indices and Fifth M-Zagreb Polynomials of PAMAM Dendrimers. Intern J Fuzzy Mathematical Archive 13(1): 99-103.
9. Munir M, Nazeer W, Rafique S, Kang S (2016) M-polynomial and degree-based topological indices of polyhex nanotubes. Symmetry 8:149.
10. Gao W, Wang W, Farahani MR (2016) Topological indices study of molecular structure in anticancer drugs. J Chem.

11. Hayat S, Imran M (2015) On degree based topological indices of certain nanotubes. J Comput Nanosci 12: 1599-1605.

12. Yang H, Rashid MA, Ahmad S, Khan SS, Siddiqui MK (2019) On molecular descriptors of face-centered cubic lattice. Processes 7: 280.

13. DH Rouvray (1987) The modeling of chemical phenomena using topological indices. Journal of computational chemistry 8: 470.

Citation: Fozia Bashir Farooq. General Fifth M-Zagreb Indices and General Fifth M-Zagreb Polynomials of Dyck-56 Network. Annal Biostat & Biomed Appli. 4(3): 2020. ABBA.MS.ID.000591. DOI: 10.33552/ABBA.2020.04.000591.