

General Fifth M-Zagreb Indices and General Fifth M-Zagreb Polynomials of Dyck-56 Network

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Abstract

A topological index is a type of molecular index that is calculated from the molecular graph of chemical structure. A topological index relates chemical structure with its underlying physical, biological and chemical properties. This article deals with General fifth M1 and General fifth M2-Zagreb indices and General fifth M1 and general fifth M2-Zagreb polynomials of Dyck-56 network.

Keywords: Dyck-56 Network; Zagreb index; Zagreb polynomial

Introduction

A chemical graph can be represented by polynomial, a numerical value or a matrix form. Topological indices are major source of the relationship between physical and biological properties and behavior of chemical structures. Topological indices are used in development quantitative structure activity relationships in which properties of molecules are correlated with their chemical structure. Let $G(V, E)$ be a graph with vertex set V and edge set E , with $|V| = m$ and $|E| = n$. Let d_u represent the degree of a vertex u in graph G . S_u stands for sum of degrees of the vertices incident with vertex u in G . Milan Randic [1] introduced the first degree-based Topological index, i.e Randic Index, Defined as;

$$X(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_u d_v}} \quad (1)$$

Furthermore, Bollabas and Erdos [2] introduced general Randic index as;

$$R_\alpha(G) = \sum_{uv \in E(G)} (d_u d_v)^\alpha \quad (2)$$

Gutman and Trinajstic [3] introduced first and second Zagreb indices as;

$$M_1(G) = \sum_{uv \in E(G)} [d_u + d_v] \quad (3)$$

$$M_2(G) = \sum_{uv \in E(G)} [d_u d_v] \quad (4)$$

Recently in [4], Kulli introduced new indices that are general fifth M_1 - Zagreb indices denoted by $M_1^\alpha G_5(G)$ and general fifth M_2 - Zagreb indices denoted by $M_2^\alpha G_5(G)$ defined as;

$$M_1^\alpha G_5(G) = \sum_{uv \in E(G)} [s_u(G) + s_v(G)]^\alpha \quad (5)$$

$$M_2^\alpha G_5(G) = \sum_{uv \in E(G)} [s_u(G) s_v(G)]^\alpha \quad (6)$$

The general fifth M_1 - Zagreb polynomial denoted by $M_1^\alpha G_5(G, x)$ and general fifth M_2 - Zagreb polynomial denoted by $M_2^\alpha G_5(G, x)$ is defined as;

$$M_1^\alpha G_5(G, x) = \sum_{uv \in E(G)} x^{[s_u(G) + s_v(G)]^\alpha} \quad (7)$$

$$M_2^\alpha G_5(G, x) = \sum_{uv \in E(G)} x^{[s_u(G)s_v(G)]^\alpha} \quad (8)$$

Dyck-56_{n×n} Networks were built up by unit representation of Dyck-56 (Dyck 1880) by identification procedure. Further detail about formation of Dyck Network can be found in [5]. Dyck graph is the 3-regular graph with 32 vertices and 48 edges, having chromatic number 2 and chromatic index 3 with diameter 5. Further

details of formation of Dyck-56_{n×n} network can be found in [5,6]. Selvan and Naranyakar in [7] calculated general Randic Index, First Zagreb index, ABC and GA indices for Dyck-x, ABC and GA indices for Dyck-56_{n×n} network.

Main Results

Figure 1 Dyck-56_{n×n}(A) Network has $12n^2 + 4n$ vertices and $18n^2 - 2n$ edges. Based on the sum of the degrees of incident vertices to the end vertices of each edge, there are seven types of edges of Dyck-56_{n×n}(A) Network. This edge partition is presented in (Table 1) [8-10].

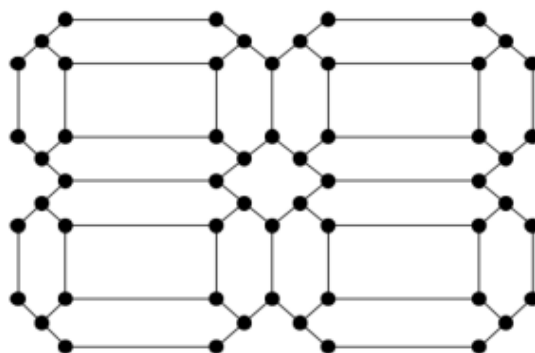


Figure 1: Dyck-562×2(A) Network.

Table 1: Edge Partition of Dyck-56_{n×n}(A) Network.

(S_u, S_v)	Frequency
(5,5)	$4n$
(5,7)	8
(5,8)	$8(n-1)$
(7,9)	4
(8,9)	$16(n-1)$
(9,9)	$18n^2 - 26n + 12$

Theorem 1: Let G be the graph Dyck-56_{n×n}(A) Network, and the general fifth M_1 -Zagreb index for G is

$$M_1^\alpha G_5(G) = 2.2^\alpha [2n.5^\alpha + 4.6^\alpha + 2.8^\alpha + (9n^2 - 13n + 6)9^\alpha + 4(n-1)(2.13^\alpha + 4.17^\alpha)]$$

Proof. Let $G =$ Dyck-56_{n×n}(A) Network; the above result can be found by using Table 1 and equation 5.

So,

$$M_1^\alpha G_5(G) = \sum_{uv \in E(G)} [s_u(G) + s_v(G)]^\alpha$$

$$= 4n[5+5]^\alpha + 8[5+7]^\alpha + 8(n-1)[5+8]^\alpha + 4[7+9]^\alpha + 16(n-1)[8+9]^\alpha + (18n^2 - 26n + 12)[9+9]^\alpha$$

$$= 4n[10]^\alpha + 8[12]^\alpha + 8(n-1)[13]^\alpha + 4[16]^\alpha + 16(n-1)[17]^\alpha + (18n^2 - 26n + 12)[18]^\alpha$$

Which further reduces to

$$M_1^\alpha G_5(G) = 2.2^\alpha [2n.5^\alpha + 4.6^\alpha + 2.8^\alpha + (9n^2 - 13n + 6)9^\alpha + 4(n-1)(2.13^\alpha + 4.17^\alpha)]$$

Theorem 2: Let G be the graph Dyck-56_{n×n}(A) Network, and the general fifth M_2 -Zagreb index for G is

$$M_2^\alpha G_5(G) = 4[n.5^\alpha + 2.7^\alpha + 2(n-1)8^\alpha]5^\alpha + 2[2.7^\alpha + 8(n-1)8^\alpha + (9n^2 - 13n + 6)9^\alpha]9^\alpha$$

Proof. Let G be Dyck-56_{n×n}(A) Network. The above result can be found using (Table 1) and equation 6.

So,

$$M_2^\alpha G_5(G) = \sum_{uv \in E(G)} [s_u(G)s_v(G)]^\alpha$$

$$= 4n[5 \times 5]^\alpha + 8[5 \times 7]^\alpha + 8(n-1)[5 \times 8]^\alpha + 4[7 \times 9]^\alpha + 16(n-1)[8 \times 9]^\alpha + (18n^2 - 26n + 12)[9 \times 9]^\alpha$$

$$= 4n[25]^{\alpha} + 8[35]^{\alpha} + 8(n-1)[40]^{\alpha} + 4[63]^{\alpha} + 16(n-1)[72]^{\alpha} + (18n^2 - 26n + 12)[81]^{\alpha}$$

Which further reduces to

$$M_2^{\alpha} G_5(G) = 4[n.5^{\alpha} + 2.7^{\alpha} + 2(n-1)8^{\alpha}]5^{\alpha} + 2[2.7^{\alpha} + 8(n-1)8^{\alpha} + (9n^2 - 13n + 6)9^{\alpha}]9^{\alpha}$$

Theorem 3: Let G be the graph Dyck-56 $_{n \times n}$ (A) Network, and the general fifth M_1 - Zagreb polynomial for G is

$$M_1^{\alpha} G_5(G, x) = 4[nx^{10\alpha} + 2x^{12\alpha} + x^{16\alpha}] + (18n^2 - 26n + 12)x^{18\alpha} + [8(n-1)\{x^{13\alpha} + 2x^{17\alpha}\}]$$

Proof. Let G be Dyck-56 $_{n \times n}$ (A) Network. The above result can be found using (Table 1) and equation 7.

So,

$$M_1^{\alpha} G_5(G, x) = \sum_{uv \in E(G)} x^{[s_u(G) + s_v(G)]^{\alpha}}$$

$$= 4nx^{15+5\alpha} + 8x^{15+7\alpha} + 8(n-1)x^{15+8\alpha} + 4x^{17+9\alpha} + 16(n-1)x^{18+9\alpha} + (18n^2 - 26n + 12)x^{19+9\alpha}$$

$$= 4nx^{10\alpha} + 8x^{12\alpha} + 8(n-1)x^{13\alpha} + 4x^{16\alpha} + 16(n-1)x^{17\alpha} + (18n^2 - 26n + 12)x^{18\alpha}$$

Which after simplification is

$$M_1^{\alpha} G_5(G, x) = 4[nx^{10\alpha} + 2x^{12\alpha} + x^{16\alpha}] + (18n^2 - 26n + 12)x^{18\alpha} + [8(n-1)\{x^{13\alpha} + 2x^{17\alpha}\}]$$

Theorem 4: Let G be the graph Dyck-56 $_{n \times n}$ (A) Network, The general fifth M_2 - Zagreb polynomial for G is

$$M_2^{\alpha} G_5(G, x) = 4[nx^{125\alpha} + x^{163\alpha}] + (18n^2 - 26n + 12)x^{181\alpha} + 8(n-1)[x^{140\alpha} + 2x^{172\alpha}]$$

Proof. Let G be Dyck-56 $n \times n$ (A) Network. The above result can be found by using (Table 1) and equation 7.

So,

$$M_2^{\alpha} G_5(G, x) = \sum_{uv \in E(G)} x^{[s_u(G) \times s_v(G)]^{\alpha}}$$

$$= 4nx^{15 \times 5\alpha} + 8x^{15 \times 7\alpha} + 8(n-1)x^{15 \times 8\alpha} + 4x^{17 \times 9\alpha} + 16(n-1)x^{18 \times 9\alpha} + (18n^2 - 26n + 12)x^{19 \times 9\alpha}$$

$$= 4nx^{125\alpha} + 8x^{163\alpha} + 8(n-1)x^{140\alpha} + 4x^{163\alpha} + 16(n-1)x^{172\alpha} + (18n^2 - 26n + 12)x^{181\alpha}$$

This after simplification is

$$M_2^{\alpha} G_5(G, x) = 4[nx^{125\alpha} + x^{163\alpha}] + (18n^2 - 26n + 12)x^{181\alpha} + 8(n-1)[x^{140\alpha} + 2x^{172\alpha}]$$

Figure 2 Dyck-56 $_{n \times n}$ (B) Network has $18n^2 - 3n$ vertices and $24n^2 - 4n + 8$ edges. On the base of sum of degrees of incident vertices to end vertices of each edge there are seven types of edges of Dyck-56 $_{n \times n}$ (B) Network. This edge partition is given in (Table 2) [11-13].

Theorem 5: Let G be the graph Dyck-56 $_{n \times n}$ (B) Network, The general fifth M_1 - Zagreb index for G is

$$M_1^{\alpha} G_5(G) = 8(n-1)[3]^{\alpha}[5^{\alpha} + 7^{\alpha}] + [2]^{\alpha+2}[2 \times 8^{\alpha} + 2n^2 9^{\alpha} + (n^2 - n)10^{\alpha}] + (4n + 8)[13]^{\alpha} + (12n^2 - 20n + 8)[23]^{\alpha}$$

Proof. Let G be Dyck-56 $n \times n$ (A) Network, The above result can be found by using (Table 1) and equation 5.

So,

$$M_1^{\alpha} G_5(G) = \sum_{uv \in E(G)} [s_u(G) + s_v(G)]^{\alpha}$$

$$= (4n + 8)[6 + 7]^{\alpha} + 8(n-1)[6 + 9]^{\alpha} + 8[7 + 9]^{\alpha} + 8n^2[9 + 9]^{\alpha} + 4n(n-1)[9 + 11]^{\alpha} + 8(n-1)[9 + 12]^{\alpha} + (12n^2 - 20n + 8)[11 + 12]^{\alpha}$$

$$= (4n + 8)[13]^{\alpha} + 8(n-1)[15]^{\alpha} + 8[16]^{\alpha} + 8n^2[18]^{\alpha} + 4n(n-1)[20]^{\alpha} + 8(n-1)[21]^{\alpha} + (12n^2 - 20n + 8)[23]^{\alpha}$$

Which further reduces to

$$M_1^{\alpha} G_5(G) = 8(n-1)[3]^{\alpha}[5^{\alpha} + 7^{\alpha}] + [2]^{\alpha+2}[2 \times 8^{\alpha} + 2n^2 9^{\alpha} + (n^2 - n)10^{\alpha}] + (4n + 8)[13]^{\alpha} + (12n^2 - 20n + 8)[23]^{\alpha}$$

Theorem 6: Let G be the graph Dyck-56 $_{n \times n}$ (B) Network. The general fifth M_2 - Zagreb index for G is

$$M_2^{\alpha} G_5(G) = 8(n-1)54^{\alpha}[1 + 2]^{\alpha} + 4 \times 9^{\alpha}[2 \times 7^{\alpha} + 2n^2 9^{\alpha} + (n^2 - n)11^{\alpha}] + (4n + 8)[42]^{\alpha} + (12n^2 - 20n + 8)[132]^{\alpha}$$

Proof. Let G denotes Dyck-56 $_{n \times n}$ (B) Network. The above result can be found by using (Table 2) and equation 6.

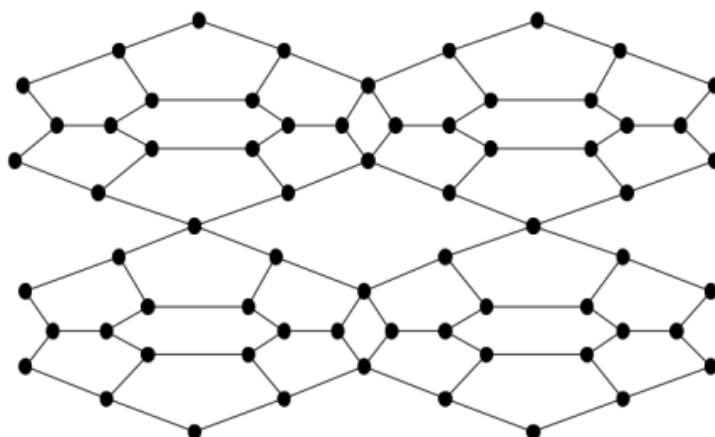


Figure2: Dyck-562x2(A) Network

Table2: Edge Partition of Dyck-56_{n×n}(B) Network.

(S_u, S_v)	Frequency
(6,7)	$4n + 8$
(6,9)	$8(n - 1)$
(7,9)	8
(9,9)	$8n^2$
(9,11)	$4n(n - 1)$
(9,12)	$8(n - 1)$
(11,12)	$12n^2 - 20n + 8$

So,

$$M_1^\alpha G_5(G) = \sum_{uv \in E(G)} [s_u(G), s_v(G)]^\alpha$$

$$= (4n + 8)[6 \times 7]^\alpha + 8(n - 1)[6 \times 9]^\alpha + 8[7 \times 9]^\alpha + 8n^2[9 \times 9]^\alpha + (4n^2 - 4n)[9 \times 11]^\alpha + 8(n - 1)[9 \times 12]^\alpha + (12n^2 - 20n + 8)[11 \times 12]^\alpha$$

$$= (4n + 8)[42]^\alpha + 8(n - 1)[54]^\alpha + 8[63]^\alpha + 8n^2[81]^\alpha + (4n^2 - 4n)[99]^\alpha + 8(n - 1)[108]^\alpha + (12n^2 - 20n + 8)[132]^\alpha$$

Which further reduces to

$$M_1^\alpha G_5(G) = 8(n - 1)54^\alpha [1 + 2]^\alpha + 4 \times 9^\alpha [2 \times 7^\alpha + 2n^2 9^\alpha + (n^2 - n)11^\alpha] + (4n + 8)[42]^\alpha + (12n^2 - 20n + 8)[132]^\alpha$$

Theorem 7: Let G be the graph Dyck-56_{n×n}(B) Network, The general fifth M_1 - Zagreb polynomial for G is

$$M_1^\alpha G_5(G, x) = (4n + 8)x^{113} + 8(n - 1)[x^{115} + x^{121}] + 4[2x^{116} + 2n^2 x^{118} + n(n - 1)x^{120} + (18n^2 - 26n + 12)x^{123}]$$

Proof. Let G is Dyck-56_{n×n}(B) Network, The above result can be found by using (Table 2) and equation 7.

So,

$$M_1^\alpha G_5(G, x) = \sum_{uv \in E(G)} x^{[s_u(G) + s_v(G)]^\alpha}$$

$$= (4n + 8)x^{16+7} + 8(n - 1)x^{6+9} + 8x^{7+9} + 8n^2 x^{9+9} + 4n(n - 1)x^{9+11} + 8(n - 1)x^{9+12} + (18n^2 - 26n + 12)x^{11+12}$$

$$= (4n + 8)x^{113} + 8(n - 1)x^{115} + 8x^{16} + 8n^2 x^{118} + 4n(n - 1)x^{20} + 8(n - 1)x^{21} + (18n^2 - 26n + 12)x^{23}$$

Which after simplification is

$$M_1^\alpha G_5(G, x) = (4n + 8)x^{113} + 8(n - 1)[x^{115} + x^{121}] + 4[2x^{116} + 2n^2 x^{118} + n(n - 1)x^{120} + (18n^2 - 26n + 12)x^{123}]$$

Theorem 8: Let G be the graph Dyck-56_{n×n}(B) Network, The general fifth M_2 - Zagreb polynomial for G is

$$M_2^\alpha G_5(G, x) = (4n + 8)x^{142} + 8(n - 1)[x^{154} + x^{108}] + 4[x^{163} + 2n^2 x^{181} + (4n + 8)x^{199}] + (18n^2 - 26n + 12)x^{132}$$

Proof. Let G be Dyck-56_{n×n}(B) Network. The above result can be found by using (Table 2) and equation 8.

So,

$$M_1^\alpha G_5(G, x) = \sum_{uv \in E(G)} x^{[s_u(G), s_v(G)]^\alpha}$$

$$= (4n + 8)x^{6 \times 7} + 8(n - 1)x^{6 \times 9} + 8x^{7 \times 9} + 8n^2 x^{9 \times 9} + (4n^2 - 4n)x^{9 \times 11} + 8(n - 1)x^{9 \times 12} + (18n^2 - 26n + 12)x^{11 \times 12}$$

$$= (4n + 8)x^{42} + 8(n - 1)x^{54} + 8x^{63} + 8n^2 x^{81} + (4n^2 - 4n)x^{99} + 8(n - 1)x^{108} + (18n^2 - 26n + 12)x^{132}$$

Which after simplification is

$$M_1^\alpha G_5(G, x) = (4n + 8)x^{42} + 8(n - 1)[x^{54} + x^{108}] + 4[x^{63} + 2n^2 x^{81} + (4n + 8)x^{99}] + (18n^2 - 26n + 12)x^{132}$$

Conclusion

Degree based topological indices like General fifth M_1 and General fifth M_2 Zagreb indices for Dyck-56 Network are found in this paper. These indices are useful in study of QSAR/QSPR. Furthermore, General fifth M_1 and General fifth M_2 -Zagreb polynomials for Dyck-56 Networks are found. These indices and polynomials are useful for the study to understand correlation between physical structures with chemical properties.

Acknowledgement

None.

Conflict of Interest

No conflict of interest.

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