Mini Review

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A New T-X Family of Distributions

Clement Boateng Ampadu*

Department of Biostatistics, USA

*Corresponding author: Clement Boateng Ampadu, Department of Biostatistics, USA.

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Abstract

This paper introduces a new weight in the T – X (W) framework, and shows a new family induced by this new weight is good in fitting real-life data.

Keywords: Exponential distribution; Normal distribution; T-X family of distributions

The T – X(W) Family of Distributions

This family of distributions is a generalization of the the beta-generated family of dis-tributions first proposed in [1] In particular, let $r(t)$ be the PDF of the random variable $T \in [a, b]$, $-\infty \leq a < b \leq \infty$, and let $W(F(x))$ be a monotonic and absolutely continu-ous function of the CDF $F(x)$ of any random variable X . The CDF of a new family of distributions defined in [2] is given by

$$G(x) = \int_a^{W(F(x))} r(t) dt = R\{W(F(x))\}$$

where $R(\cdot)$ is the CDF of the random variable T and $a \geq 0$

Remark 3.1: The PDF of the T – X(W) family of distributions is obtained by differentiating the CDF above

Remark 3.2. When we set $W(x) := -\log(1-x)$, then we use the term “T-X” to de-scribe all sub-classes of the T-X(W) family of distributions induced by the weight function $W(x) = -\log(1-x)$. A description of different weight functions that are appropriate given the support of the random variable T is discussed in [2]

A plethora of results studying properties and application of the T-X(W) family of distri-butions have appeared in the literature, and the research papers, assuming open access, can be easily obtained on the web via common search engines, like Google, etc

The $\left(\frac{1}{e}\right)^\alpha$ PT-G Family of Distributions

Definition 4.1: [3] Let $\alpha \neq \frac{1}{e}$, $\alpha > \frac{1}{e}$ and $\xi > 0$ where α is the rate scale parameter and ξ is a vector of parameters in the baseline distribution all of whose entries are positive. A random variable Z is

said to follow the $\left(\frac{1}{e}\right)^\alpha$ power transform family of power transform family of distributions if the Cumulative Distribution Function (CDF) is given by

$$F(x; \alpha, \xi) = \frac{1 - e^{-\alpha G(x; \xi)}}{1 - e^{-\alpha}} \quad x \in \mathbb{R},$$

where the baseline distribution has CDF $G(x; \xi)$.

Proposition 4.2: [3] The PDF of the $\left(\frac{1}{e}\right)^\alpha$ power transform family distribution is given by

$$f(x; \alpha, \xi) = \frac{1}{1 - e^{-\alpha}} \left(\alpha g(x, \xi) e^{-\alpha G(x; \xi)} \right)$$

Where $\alpha \in \mathbb{R}$ and $\alpha \neq 0$ and $x \in [0, 1]$ and $G(x; \xi)$ is the baseline cumulative distribution with probability density function $g(x; \xi)$.

The New Weight and the Distribution

By modifying the parameter space for α in the $\left(\frac{1}{e}\right)^\alpha$ PT-Standard Uniform family and using it in Remark 1.2, we define the new weight as

$$W^*(x; \alpha) = -\log\left(\frac{1 - e^{\alpha - \alpha x}}{1 - e^{-\alpha}}\right)$$

where $\alpha \in \mathbb{R}$ and $\alpha \neq 0$ and $x \in [0, 1]$. Using this we introduce the following

Definition 5.1: Let X be a random variable with CDF $F(x; \xi)$ and PDF $f(x; \xi)$; and let T be a random variable with support $[0, \infty)$ whose PDF is $r(t; \gamma)$, and whose CDF is $R(t; \gamma)$; let ξ and γ be vector of parameters in the distribution of the random variables X and T ,

respectively. The new T-X family distributions has CDF defined by the following

$$G(x; \xi, \gamma, \alpha) = \int_0^{\log\left(\frac{1-e^{-\alpha F(x; \xi)}}{1-e^{-\alpha}}\right)} r(t; \gamma) = R\left[-\log\left(\frac{1-e^{-\alpha F(x; \xi)}}{1-e^{-\alpha}}\right); \gamma\right] \quad 3.1$$

where, $\alpha, x \in \mathbb{R}$, and $\alpha \neq 0$

Remark 5.2: The PDF of the new T-X family of distributions can be obtained by differentiating the CDF above

Practical Illustration

Assume X is a normal random variable with CDF

$$F(x; c, d) = \frac{1}{2} \operatorname{erfc}\left(\frac{c-x}{\sqrt{2d}}\right)$$

Where $c \in \mathbb{R}, d > 0$ and $\operatorname{erfc}(z) = 1 - \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt$. Also assume the random variable T is an exponential random variable with CDF

$$R(t; b) = 1 - e^{-bt}$$

where $t, b > 0$. Now the following is immediate from Definition

Proposition 6.1: The CDF of the new Exponential-Normal family of distributions is given by

$$G(x; c, d, b, \alpha) = 1 - \left(\frac{e^{\frac{1}{2}\alpha} \operatorname{erfc}\left(\frac{c-x}{\sqrt{2d}}\right) + 1}{e^\alpha - 1} \right)^b$$

where $c, \alpha \in \mathbb{R}, b, d > 0, \alpha \neq 0$, and $\operatorname{erfc}(z) = 1 - \operatorname{erf}(z) = 1 - \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt$.

Remark 6.2: If a random variable M has CDF given by the previous Proposition we write $M \sim \text{NEN}(c, d, b, \alpha)$ (Figure 1 & 2) [4]

The fit in Figure 1 and the approximately straight line in Figure 2 indicates the NEN distribution is a good fit.

Figure 1: The survival function of NEN(3.11779, 1.0106, 1.62634, -0.184163) fitted to the empirical (survival) distribution.

Figure 2: The probability plots of 1500 random samples of NEN(3.11779, 1.0106, 1.62634, -0.184163).

By differentiating the CDF, we have the following

Proposition 6.3: The PDF of the new Exponential-Normal family of distributions is given by

where

$$c, \alpha \in \mathbb{R}, b, d > 0, \alpha \neq 0, \text{ and } \operatorname{erfc}(z) = 1 - \operatorname{erf}(z) = 1 - \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt.$$

(Figure 3 & 4)

Figure 3: The box-whisker charts of 1500 random samples of $NEN(3.11779, 1.0106, 1.62634, -0.184163)$.

Figure 4: The quantile plots of 1500 random samples of $NEN(3.11779, 1.0106, 1.62634, -0.184163)$.

As the box-whisker charts in Figure 3 and the quantile plots in Figure 4 appear almost identical, we have more evidence that the NEN distribution is a good fit.

Concluding Remarks

This paper has introduced a new T-X family of distributions via Definition 3.1 and has shown a member of this new class of distributions is a good fit to real life data. Our hope is that the researchers of distribution theory and its applications will further develop the properties and applications of the new T-X class of distributions.

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Conflict of Interest

No conflict of interest.

References

1. Eugene N, Lee C, Famoye F (2002) The beta-normal distribution and its applications. *Communications in Statistics-Theory and Methods* 31(4): 497-512.
2. Alzaatreh A, Lee C, Famoye F (2013b) A new method for generating families of continuous distributions. *Metron* 71(1): 63-79.
3. Abdulzeid Yen Anafo (2019) The New Alpha Power Transform: Properties and Applications, Master of Science in Mathematical Sciences Essay, African Institute for Mathematical Sciences (Ghana). Unpublished Manuscript.
4. Ayman Alzaatreh, Carl Lee Felix Famoye (2014) T-normal family of distributions: a new approach to generalize the normal distribution, *Journal of Statistical Distributions and Applications* 1:16.