

Opinion

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The Cot-X Family of Distributions with Application

Clement Boateng Ampadu*

Department of Biostatistics, USA

*Corresponding author: Clement Boateng Ampadu, Department of Biostatistics, USA.

Received Date: August 18, 2019
Published Date: August 20, 2019

Abstract

We modify the cotangent function into a new statistical distribution and show its applicability.

Keywords: Trigonometric functions; Statistical distributions; Breast cancer; Odds T-X Contents

Introduction

Statistical distributions arising from trigonometric functions have populated the literature, and for example, see [1-4]. On the other hand, the T-X(W) family of distributions appeared in [5], and in the special case the random variable T has support [0, ∞), and the weight function is given by $w(x) = \frac{x}{1-x}$, we get the so-called Odds T-X family of distributions with the following integral representation for its CDF

$$\int_0^{F(x)} \frac{r(t)}{1-F(x)} dt$$

where the random variable X has CDF F(x) and the random variable T has PDF r(t).

By these observations, this paper unfolds as follows. We introduce a so-called Cot-X family of distributions in Section 2, and in Section 3 we introduce a so called Cot Odds T-X family of distributions. Section 4 and Section 5, show applicability of the new families, and the last section is devoted to the conclusions.

The New Family

The CDF of Cot-X is defined as

$$G(x; \xi) = -\cot\left(\frac{1}{4}\pi(F(x; \xi) + 2)\right)$$

where $x \in \mathbb{R}$, the baseline distribution has CDF F(x; ξ) and PDF f(x; ξ). By differentiating the CDF, the PDF of Cot-X is given as

$$g(x; \xi) = \frac{1}{4}\pi f(x; \xi) \csc^2\left(\frac{1}{4}\pi(F(x; \xi) + 2)\right)$$

A New Variant of the Odds T-X Family of Distributions

Let T be a random variable with support [0, ∞), whose PDF and CDF are given by r(t; ξ) and R(t; ξ), respectively, with ξ being a vector of parameters in the distribution of T, and let X be a random variable with PDF f(x; β) and CDF F(x; β), where β is a vector of parameters in the distribution of X. We define the Cot Odds T-X family of distributions with the following integral for its CDF

$$k(x; \beta, \xi) = \frac{1}{4}\pi \int_0^{F(x; \beta)} \frac{r(t; \xi)}{1-F(x; \beta)} \csc^2\left(\frac{1}{4}\pi(R(t; \xi) + 2)\right) dt,$$

$$x \in \mathbb{R}$$

Explicitly the CDF of the Cot Odds T-X family is given by

$$k(x; \beta, \xi) = -\cot\left(\frac{1}{4}\pi\left(R\left(\frac{F(x; \beta)}{1-F(x; \beta)}; \xi\right) + 2\right)\right)$$

and the PDF is given by

$$k(x; \beta, \xi) = \frac{1}{4}\pi \csc^2\left(\frac{1}{4}\pi\left(R\left(\frac{F(x; \beta)}{1-F(x; \beta)}; \xi\right) + 2\right)\right) r\left(\frac{F(x; \beta)}{1-F(x; \beta)}; \xi\right) \frac{f(x; \beta)}{(F(x; \beta) - 1)^2}$$

Practical Illustration of Cot-X

We assume X is a Dagum random variable with CDF

$$F(x; a, b, c) = \left(\left(\frac{x}{c}\right)^{-b} + 1\right)^{-a}$$

for $x, a, b, c > 0$. Now from Section 2 we have the following

Proposition 6.1. The CDF of Cot-Dagum is given by



$$G(x; a, b, c) = -\cot \left(\frac{1}{4} \pi \left(\left(\left(\frac{x}{c} \right)^{-b} + 1 \right)^{-a} + 2 \right) \right)$$

where $x, a, b, c > 0$

Remark 6.2. The PDF of Cot-Dagum can be obtained by differentiating the CDF. We write $S \sim CD(a, b, c)$, if S is a Cot-Dagum random variable (Figure 1).

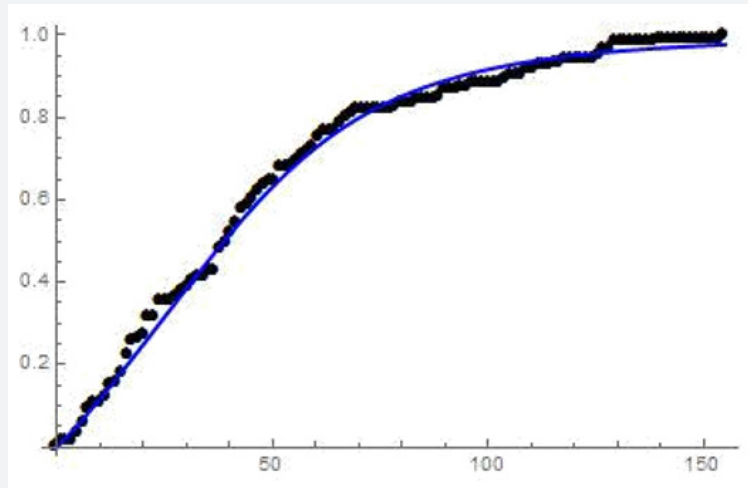


Figure 1: The CDF of $CD(0.337462, 3.19503, 58.7951)$ fitted to the empirical distribution of the data on patients with breast cancer, Section 5.2 [6].

Practical Illustration of Cot Odds T-X

We assume T is a Frechet random variable with CDF

$$R(t; c, d) = e^{-\left(\frac{t}{d}\right)^{-c}}$$

for $x, c, d > 0$. We also assume that X is a Weibull random variable with CDF given by

$$F(x; a, b) = 1 - e^{-\left(\frac{x}{b}\right)^a}$$

for $x, a, b > 0$. Now from Section 3, we have the following

Proposition 7.1. The CDF of Cot Odds Frechet-Weibull is given by

$$K(x; a, b, c, d) = -\cot \left(\frac{1}{4} \pi e^{\left(\frac{\left(\frac{x}{b} \right)^a \left(1 - e^{-\left(\frac{x}{d} \right)^{-c}} \right)^{-c}}{d} \right)} + 2 \right)$$

where $x, a, b, c, d > 0$

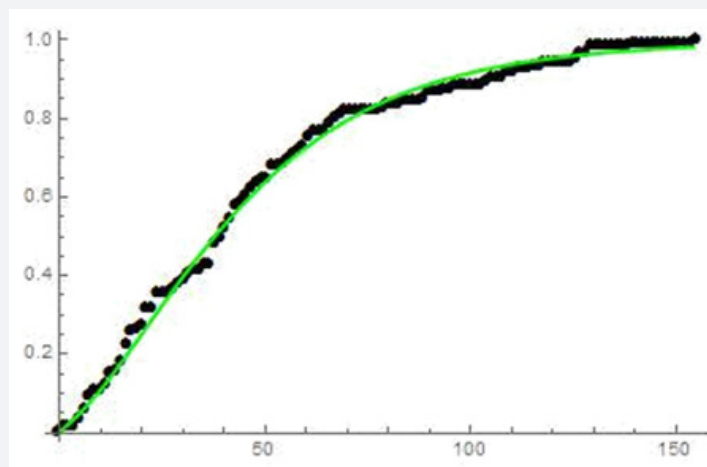


Figure 2: The CDF of $COF W(0.73884, 1.47816, 0.192653, 2062.69)$ fitted to the empirical distribution of the data on patients with breast cancer, Section 5.2.

Remark 7.2. The PDF of Cot Odds Frechet-Weibull can be obtained by differentiating the CDF. We write $J \sim COF W(a, b, c, d)$, if J is a Cot Odds Frechet-Weibull random variable (Figure 2).

Concluding Remarks

In the present paper we introduced the Cot-X and Cot Odds T-X family of distributions respectively and have shown fit to real-life

data. The future interesting problem is to investigate some properties and application of these new classes of statistical distributions.

Acknowledgement

None.

Conflict of Interest

No conflict of interest.

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