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Short Communication

The $r_T - X$ Family of Distributions induced by V: A New Method for Generating Continuous Distributions with Illustration to Cancer Patients Data

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Abstract

In this short note we introduce a new technique to generate continuous distributions which is partially inspired by [1] and [2]. An illustration to cancer patients data is also given, indicating the new class of distributions will be useful in modeling and forecasting data in various disciplines.

Keywords: Quantile generated probability distributions; Cancer patients data, T – X(W) family of distributions

Introduction

At first we recall the $q_T - X(V)$ framework which is inspired by [1] and [2]

Definition 3.1. Let V be any function such that the following holds:

a) $F(x) \in [V(a), V(b)]$

b) $F(x)$ is differentiable and strictly increasing

c) $\lim_{x\to-\infty} F(x) = V(a)$ and $\lim_{x\to-\infty} F(x) = V(b)$

then the CDF of the $q_T - X(V)$ family induced by V is given by

$$K(x) = \int_a^{V(F(x))} \frac{1}{r(Q(t))} dt$$

where $\frac{1}{r(Q(t))}$ is the quantile density function of random variable

$T \in [a,b]$, for $-\infty \le a < b \le \infty$, and F(x) is the CDF of any random variable X.

Theorem 3.2. The CDF of the $q_T - X$ family induced by V is given by

$$K(x) = Q[V(F(x))]$$

Proof. Follows from the previous definition and noting that $Q' = \frac{1}{roQ}$

Theorem 3.3. The PDF of the $q_T - X$ family induced by V is given by

$$K(x) = \frac{f(x)}{r[Q(V(F(x)))]} V'[F(x)]$$

Proof $k = K', Q' = \frac{1}{roQ}, F' = f$ and K is given by Theorem 3.2

Remark 3.4. When the support of T is $[a, \infty)$, where $a \ge 0$, we can take V as follows

a) $V(x) = 1 - e^{-x}$

b) $V(x) = \frac{x}{1+x}$

c) $V(x) = [1 - e^{-x}]^{\frac{1}{\alpha}}$ where $\alpha > 0$

d) $V(x) = \left[\frac{x}{1+x}\right]^{\frac{1}{\alpha}}$ where $\alpha > 0$

Remark 3.5. When the support of T is $(-\infty, \infty)$, we can take V as follows

a) $V(x) = 1 - e^{-e^{-x}}$

b) $V(x) = \frac{e^x}{1+e^x}$

c) $V(x)=\left[1-e^{-e^{-x}}\right]^{\frac{1}{\alpha}}$ where $\alpha>0$

d) $V(x)=\left[\frac{e^x}{1+e^x}\right]^{\frac{1}{\alpha}}$ where $\alpha>0$

Motivation for the New Family

Suppose a random variable T has quantile function Q, quantile density q, CDF R, and PDF r, then since

$$Q=R^{-1}$$

The following is clear

$Q(R(t))=t$ and $R(Q(t))=t$

If we differentiate both sides of, $R(Q(t))=t$, with respect to t and solve for q(t), we get the integrand in Definition 4.1. On the other hand, if

we differentiate both sides of, $Q(R(t))=t$,with respect to t, and solve for r(t), we can see that $r(t)=\frac{1}{q(R(t))}$, thus making this replacement to the integrand

in Definition 1.1 leads to the following

Definition 4.1. Let V be any function such that the following holds:

a) $F(x)\in[V(a),V(b)]$

b) $F(x)$ is differentiable and strictly increasing

c) $\lim_{x\to-\infty}F(x)=V(a)$ and $\lim_{x\to-\infty}F(x)=V(b)$

then the CDF of the r_T-X family induced by V is given by

$$J(x)=\int_a^{V(F(x))}\frac{1}{q(R(t))}dt$$

where $\frac{1}{q(R(t))}$ is the probability density function of random variable $T\in[a,b]$, for] $-\infty\le a<b<\infty$, and F(x) is the CDF of any random variable X.

Notation 4.2. Ψ will denote the class of all functions V satisfying (a), (b), and (c) in the definition immediately above

The CDF of the New Family

Since $\frac{dR(t)}{dt}=r(t)=\frac{1}{q(R(t))}$, then the definition immediately above

implies the following

Theorem 5.1. The CDF of the r_T-X family induced by V is given by

$$J(x)=R(V(F(x)))$$

where the random variable $T\in[a,b]$, for $-\infty\le a<b<\infty$, has CDF $R,V\in\Psi$, and F(x) is the CDF of any random variable X.

The PDF of the New Family

By differentiating the CDF in Theorem 5.1, with respect to x, we have the following

Theorem 6.1. The PDF of the r_T-X family induced by V is given by

$$j(x)=r(V(F(x)))V'(F(x))f(x)$$

where the random variable $T\in[a,b]$, for $-\infty\le a<b<\infty$, has PDF $r,V\in\Psi,F(x)$ and f(x) are the CDF and PDF, respectively, of any random variable X.

Practical Significance

At first we introduce the following

Definition 7.1**.** A random variable K will be said to follow the new r_T-X family of distributions of type I, if the CDF can be expressed as the following integral

$$J_1(x)=\int_0^{\frac{F(x)-\mathrm{Pr}\,oductLog(F(x))}{F(x)}}\frac{1}{(q_T oR_T)(t)}dt$$

where the random variable X has CDF F(x), and the random variable T with support $[0,\infty)$ has quantile density qT and CDF RT, and Product $Log[z]$ gives the principal solution for w in $z=we^w$

Remark 7.2. Note that $V(x)=\frac{x-\mathrm{Pr}\,oductLog(x)}{x}$ is our weight in the above definition, and

V^{-1} gives $\frac{-\log(1-x)}{1-x}$ which is the weight function introduced in [3]

Since $R'=\frac{1}{qoR}$, then the above definition implies the following

Theorem 7.3. The CDF of the new r_T-X family of distributions of type I is given by

$$J_1(x)=R_T\left(\frac{F(x)-\mathrm{Pr}\,oductLog(F(x))}{F(x)}\right)$$

where the random variable T with support [0,1) has CDF R_T, Product Log[z] gives the principal solution for w in $z=we^w$, and the random variable X has CDF F(x)

Remark 7.4. By differentiating the CDF above, the PDF of the new r_T-X family of distributions of type I can be obtained

By assuming $F(x):=1-e^{-\left(\frac{x}{b}\right)^a}$, that is, X is Weibull distributed and $R_T(t)=1-e^{-cx}$, that is, T is exponentially distributed. We get the following from Theorem 7.3

Theorem 7.5. The CDF of the new Exponential-Weibull family of distributions of type I is given by

$$J_1(x;a,b,c)=1-\exp\left(-\frac{c\left(\mathrm{Pr}\,oductLog\left(1-e^{-\left(\frac{x}{b}\right)^a}\right)-e^{-\left(\frac{x}{b}\right)^a}+1\right)}{1-e^{-\left(\frac{x}{b}\right)^a}}\right)$$

where a, b, c, x > 0, and Product Log[z] gives the principal solution for w in $z=we^w$

Remark 7.6. When a random variable B has CDF given by the new Exponential-Weibull family of distributions of type I we write $B\sim NEW(a,b,c)$ (Figure 1 &2).

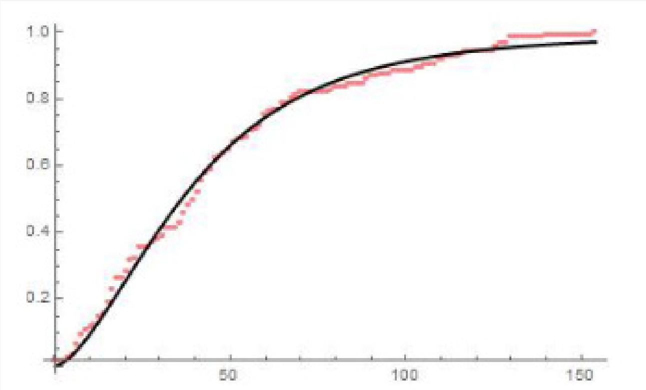

Figure 1: The CDF of NEW (1.58547, 208.317, 12.5369) fitted to the empirical distribution of the data on patients with breast cancer, Section 5.2 [4].

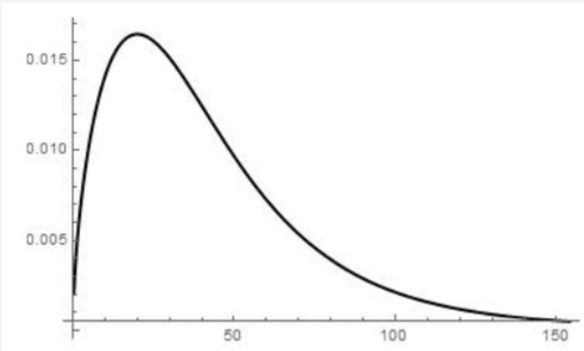

Figure 2: The PDF of NEW (1.58547, 208.317, 12.5369) over [0.3, 154].

Remark 7.7. The PDF of the new Exponential-Weibull family of distributions of type I can be obtained by differentiating the CDF above.

Acknowledgement

None.

Conflict of Interest

No conflict of interest.

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