Research Article

Optimal Designs for M-Allel Crosses in One and Two-Way Setting

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Abstract

Recently authors [1] constructed optimal Block and Row-Column designs for complete diallel cross (CDC) methods by using nested balanced incomplete block designs.In this article we are presenting methodof constructionof three series of optimal block and row-column designs for partial double cross by usingthese designs. Ourproposed optimal block and row-column designs for partial double cross are new and not available in statistical literature.AMS classification: 62k 05

Keywords: Nested balanced incomplete block design; Complete diallel cross;Partal double cross; Row and Column design; Block design

Introduction

Mating design involving multi–allele cross m (≥2) lines play very important role to study the genetic properties of a set of inbred lines in plant breeding experiments. A most common mating design in genetics is the diallel cross which consist ofv=p(p-1)/2 crosses of p inbred lines such that the crosses of the type (i × j) = (j × i) for i,j = 1,2, ..., p.This type of mating design is called complete diallel cross (CDC) method 4 of Griffing[2]. The concept of CDC can be easily extended to double cross designs. A double cross design is obtained by crossing two unrelated F_1 hybrids symbolized as (i ×j) and (k × l), where i ≠j≠k ≠l ≠i, are 4 parents and (i× j) and (k × l) are two F_1's[3].

Let n_c denote the total number of crosses (experimental units) involved in an m-allele cross experiment, where m=2 or 4. Generally double cross experiments are conducted using a completely randomized design (CRD) or a randomized complete block (RCB) design involving some or all n_c crosses as treatments. The number of crosses in such a mating design increases rapidly with increase in the number of lines. It leads to an overall inefficient experiment. It is for this reason that the use of incomplete block design as environment design is needed for double cross experiments [3].

Parsad et al. [4] constructed optimal block designs for double cross experiments by using balanced incomplete block designs and nested balanced incomplete block designs of Morgan et al. [5].

Sharma & Tadesse [6] constructed double cross designs for even and odd value of p by using initial block of unreduced balanced incomplete block designsgiven by Bose et al. [7] and initial block of row –column designs given byGupta & Choi [8], respectively.

In this paper we have used nested balanced incomplete block designs of Dey et al. [9] and Morgan et al. [5] for the construction of three seriesof optimal block and row-column designs for partial double cross experiment. The parameters of ourproposed optimal block and row-column designs for partial double cross experiments are different from Parsad et al. [4] designs. We have used those designs of Morgan et al [5] for the construction of partial double cross experiment which Parsadet al. [4] did not use.In our proposed designs every cross is replicated equal number of times in a design. We have considered the model that includes the gca effects, apart from block effects, but no specific combining ability effects.

Some definitions

a.   Definition 2.1: The double cross has been defined by Rawlings and Cockerham (1962 b) as a cross between two unrelated F_1 hybrids, say denoted by (i ×j) and (k× l), wherei ≠j ≠k ≠ l ≠i, are denoting the grandparents and no two of them are same. Ignoring reciprocal crosses, with p grandparents, there will be $3\binom{p}{4}$ double crosses.

b. Definition 2.2:An arrangement ofv treatments each replicatedr* times in two systems of blocks is said to be a nested balanced incomplete design (NBIBD) with parameters (v, b_1, k_1, r*, λ_1, b_2, k_2, λ_2, m) if

• The second system is nested within the first, with each block from the first system, containing exactly m blocks from the second system (sub blocks);

• ignoring the second system leaves a balanced incomplete block design with parameters v, b_1, k_1, r*, λ_1;

• ignoring the first system leaves a balanced incomplete block design with parametersv, b_2, k_2, r*, λ_2.

The following parametric relations hold for a nested balanced incomplete block design:

(i)   v r*= b_1 k_1=m b_1 k_2=b_2 k_2

(ii)   (v-1) λ_1= (k_1 -1) r*

(iii)   (v-1) λ_2= (k_2 -1) r*

Universal optimality of designs for 1-way- heterogeneity

Following Parsad et al. [4], let dbe a block design for an m-allel cross experiment involving p inbred lines, b blocks each of size k. This means that there are k crosses in each of the blocks of d. Further,let $r_{dt}$and s_{di} denote the number of replication of the tth cross and the number of replications of the ith line in different crosses,respectively, in d [t = 1,2, …, n_c; i = 1,2,…,p] . Evidently, $\sum r_t$= bk, $\sum s_i$= mbk = mn and n = bk, the total number of observations. We also considered this and we took the following additive model for the observations obtained from design d.

$$y = m\mathbf{1_n} + \Delta_1' \mathbf{g} + \Delta_2' \mathbf{b} + \mathbf{e} \quad \text{.......(3.1)}$$

where yis an n×1 vector of observations, 1is an n×1 vector of ones, Δ_1' is an n × p design matrix for lines Δ_2' and is an n × b design matrix for blocks, that is, the (h,l)thelement of Δ_1' (also of Δ_2') is 1 if theh^{th} observation pertains to the l^{th}line (also of block) and is zero otherwise. μ is a general mean, g is a p × 1 vector of line parameters, β is a b × 1 vector of block parameters and e is an n × 1 vector of residuals. It is assumed that vector βis fixed and e is normally distributed with mean 0andVar (e) = σ^2IwithCov (β, e)= 0, where Iis the identity matrix of conformable order.

For the analysis of proposed design d, the method of least squares leads to the following reduced normal equations for estimating the linear functions of the gca effects of lines under model (3.1).

$$\mathbf{C_d} = \mathbf{G_d} - \mathbf{N_d}\mathbf{K_d^{-1}}\mathbf{N}_d' = (c_{ij}) \text{ (i,j= 1,2,…, p)(3.2)}$$

where G_d = Δ_1 Δ_1' = ($g_{dii'}$), g_{dii} = $s_{di}$and for i≠i′ ,$g_{dii'}$ is the number of crosses in d in which the linesi and i′ appear together . $N_d = \Delta_1$ Δ_2' = (n_{dij}), $n_{dij}$is the number of times the line i occurs in block jof dand

$K_d = \Delta_2 \Delta_2'$ is the diagonal matrix of block sizes.

A design d will be called connected if and only if rank (C_d) = p -1, or equivalently, if and only if all elementary comparison among gca effects are estimable using d. We denote byD(p,b,k), the class of all such connected block design {d} with p lines, b blocks each of size k. To prove optimality of design d,we need the following well known lemma [10].

Lemma 3.1 For given positive integers s and t, the minimum of $\sum_{i=1}^{s} n_i^2$ subject to $\sum_{i=1}^{s} n_i = t$, where n_i's are non-negative integers, is obtained when t − s [t/s] of the n_i's are equal to [t/s]+1 and s - t + s[t/s], where [z] denotes the largest integer not exceeding z. The corresponding minimum of

$$\sum_{i=1}^{s} n_i^2 \text{ is t (2[t/s] +1) – s [t/s]([t/s] +1)}$$

Theorem 3.1:For any design d ε D (p, b, k)

tr (C_d)≤k^{-1}b{m k (k -1-2x) + px (x+1)}

where x= [mk/p] and for a square matrix A, tr (A) stands for the sum of the diagonal elements of A.

Proof.For any design d ε D (p, b, k), we have

$$tr(\mathbf{C_d}) = \sum_{i=}^{p} s_{di} - k^{-1}\sum_{i=1}^{p}\sum_{j=1}^{b} n_{dij}^2 = mb\,k - k^{-1}\sum_{i=1}^{p}\sum_{j=1}^{b} n_{dij}^2$$

Now, since $\sum_{i=1}^{p}\sum_{j=1}^{b} n_{dij}^2 = mbk$, using Lemma 3.1

$$\sum_{i=1}^{p}\sum_{j=1}^{b} n_{dij}^2 \geq b\{m\,k(2x+1) - px(x+1)\} \text{ where [mk/p].}$$

Hence

tr(C_d) $\leq$ mbk– k^{-1}b{m k(2x +1) – p x (x+1)}

= k^{-1}b {m k (k-1 -2 x) + p x (x+1)}

By Lemma 3.1, the above equality is attained if and only if n_{dij}= x or x+1, for i =1,2,. . ., p; j = 1, 2,. . ., b

Theorem 3.1:For any design d ε D (p, b, k)

tr (C_d)≤ k^{-1}b{ mk (k -1-2x) + p x (x+1)

where x = [mk/p] and for a square matrix A, tr (A) stands for the sum of the diagonal elements of A.

Kiefer [11] showed that a design is universally optimal in a relevant class of competing design if (i) the information matrix (the C_d– matrix) of the design is completely symmetric in the sense that C_d has all its diagonal elements equal and all of its off-diagonal elements equal, and (ii) the matrix C_d has maximum trace in the class of competing designs, that is, such a design minimize the average variance of the best linear unbiased estimators of all elementary contrasts among the parameters of interest i.e. the general combining ability in our context. On the basis of the theorem 3.1 and Kiefer criterion of the universal optimality, we can state Theorem 3.2.

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Theorem 3.2: For any design d* ε D (p, b, k) be a block design for m-allel crosses satisfying

(i)   tr (C_{d*}) = k^{-1} b{ mk (k -1-2x) + p x (x+1)

(ii) C_{d*} = $(p-1)^{-1}k^{-1}$ b {m k (k -1-2x) + p x (x+1)} (Ip –1p 1′p/ p)

is completely symmetric.

where Ip is an identity matrix of order p and 1p1′p is a p × p matrix of all ones. Furthermore, using d* all elementary contrasts among gcaeffects are estimated with variance

$[2b^{-1}(p-1)k/ \{m k (k-1-2x) + p x(x+1)\}] \sigma^2$.

Then d* is universally optimal in D (p, b, k), and in particular minimizes the average variance of the best linear unbiased estimator of all elementary contrasts among the general combining ability effects.

Universal optimality of designs for 2-way- heterogeneity

Consider an m-allel cross involving p lines in a row-column design with k rows and b columns. The model can be written as given below

$$\mathbf{Y} = \mu\mathbf{1_n} + \Delta_1'\tau + \Delta_2'\beta + \Delta_3'\gamma + \mathbf{e} \quad (4.1)$$

where Yis an n × 1 vector of observed responses, µ is the general mean, τ, β and γ are the column vectors of pgcaparameters, krow effects and b column effects, respectively, $\Delta_1' (n\times p), \Delta_2' (n\times k), \Delta_3' (n\times b)$ are the corresponding design matrices, respectively and $\mathbf{e}$ denotes the vector of independent random errors having mean 0 and covariance matrix $\sigma^2 \mathbf{I}_n$.

Let $N_{d1} = \Delta_1\Delta_2'$ be ap × k incidence matrix of lines versus rows and $N_{d2} = \Delta_1\Delta_3'$ be a p × b incidence matrix of treatments versus columns and $\Delta_2\Delta_3' = \mathbf{1}_k\mathbf{1}_b$. Let r_{dl}denote the number of times the lth cross appears in the design d,l = 1, 2,...,v and similarlys_{di} denote the number of times the ith lineoccurs in thedesign d, i = 0, 1, . . . p-1. Under (4.1), it can be shown that the reduced normal equations for estimating the treatment effects, after eliminating the effect of rows and columns, are

$$C_d\hat{\tau} = Q \quad (4.2)$$

where $C_d = G_d - \frac{1}{b}N_{d1}N_{d1}' - \frac{1}{k}N_{d2}N_{d2}' + \frac{s_d s_d'}{s_d'1}$

Q = T – 1/b N_{d1}R – 1/k N_{d2}C + (G/bk) $s_{d'}$

$G_d = \Delta_1\Delta_1' = (g_{dii'}), N_{d1} = (n_{dij..}), n$.

is the number of times the line i occurs in row jof d, $N_{d2} = (n_{di.t}), n_{i..t}$ is the number of times cross ioccurs in column t.Q is a p × 1 vector of adjusted treatments (crosses) total, T is a p × 1 vector of treatments (lines) totals, R is a k × 1 vector of rows totals, Cis a b ×1 vector of columns totals, and G is a grand total of all observations.

We use the following theorems ofPrasadet al. [4] to prove optimality of the proposed designs.

Theorem 4.1: For any design d ε D_1(p, b, k), trace (C_d)will be maximum when

(i) $n_{dij..} = \frac{n_{di..}}{k}$ ie., the design is orthogonal with respect to the lines vs rows as blocks classification or a row –regular setting with respect to lines.

(ii) $n_{di.t} = x or x +1$ where $x = int\left(\frac{mk}{p}\right)$ ie., the lines appear either x or x=1 time in columns as blocks classification, where m = 2 or 4.

Theorem 4.2:Let d* ε D_1(p, b, k) be a row – column design satisfying

a.   trace (C_{d*}) = k^{-1}b{m k(k-1-2x) + p x (x+1)

b. $C_{d*}$is completely symmetric.

Then d* is universally optimal in D_1 (p, b, k)

Now we will show a connection between optimal block and row-column design for optimal partial double cross design with nested balanced incomplete block designs of Preece (1967).Consider now a NBIB design d obtained by developing mod (v) $t_1$initial blocks, each sub-divided into t_2 sub-blocks. The parameters of such an NBB design are v= p, b_1= vt_1, b_2= $b_1 t_2$, k_1= $2t_2$, r*, k_2 =2. If we identify the treatments of d as lines of a diallel experiment and perform double crosses among the lines appearing in the same sub block of dand arrange these sub-blocks into one bigger block and develop mod (v), we get a design d* for a double cross experiment involving p lines with $b_2$crosses arranged in b = b_1/2 blocks.Each double cross is replicated once. Such a design d* belongs toD (p, b, k). For such a design n_{d*ij} = 0 or 1for i = 1, 2, . . . ,p, j = 1, 2, . . . , b. and

C_{d*}= $(p-1)^{-1}$[(p-1) (p-5)I_p –(3p-7)$1_p 1_p'$](4.3)

Clearly C_{d*} given by (4.3) is completely symmetric and tr (C_{d*}) = p (p-5) which equals the equality given in theorem 4.2:Thus the design d* is optimal in D_1 (p, b, k) and using d* each elementary contrast among gca effects is estimated with a variance

2 σ^2/ p (p-5)(4.4)

If p is even then the NBIB design has the following parameters.

v= p, b_1= (v-1) t_1, b_2= $rb_1 k_2$, k_1= 2r k_2, r*, k_2=2., where r is the numberof replication of double cross. If we perform the same procedure given above, we get a design d* for a partial double cross experiment involving p lines with b_2 crosses arranged in b = b_1/$t_1$blocks. Such a design d*εD (p, b, k); also, for such a design n_{d*ij}= 1 or 2for i = 1, 2, . . . ,p, j = 1, 2, . . . , b. and

C_{d*} = $(r t_1)^{-1}$[r^2 (p-1) (t_1-1)I_p – r^2 (p- t_1-1)$1_p 1_p'$](4.5)

Clearly C_{d*} given by (4.5) is completely symmetric and tr (C_{d*}) = p (p-5) which equals the equality given in theorem 3.2. Thus, the design d* is optimal in D (p, b, k) and using d* each elementary contrast among gca effects is estimated with a variance

2 σ^2 r(p-1) (t_1-1)/ t_1(4.6)

Hence, we state the following theorems.

Theorem 4.3: The existence of a nested balanced incomplete block design d with parameters

$v = p$, $b_1 = vt_1$, $b_2 = b_1t_2$, $k_1 = 2t_2$, r^*, $k_2 = 2$, where p is odd, implies the existence of optimal block and row-column designs for partial double cross.

Theorem 4.4: The existence of a nested balanced incomplete block design d with parameters

$v = p$, $b_1 = (v-1)\ t_1$, $b_2 = rb_1\ k_2$, $k_1 = 2r\ k_2$, r^*, $k_2 = 2$, where p is even, implies the existence of optimal block design for partial double cross.

Method of Construction

Series 1: Let $p = 4m+1$, $m \geq 1$ be a prime or a prime power and x be a primitive element of the GF (p). Consider the following m initial blocks.

$$\left\{\left(x^i, x^{i+2m}\right), \left(x^{i+m}, x^{i+3m}\right)\right\}, i = 0, 1, 2, \ldots m-1$$

As shown by Dey et al. [9], these initial blocks, when developed in the sense of Bose [7], give rise to a nested balanced incomplete block design with parameters $v = p = 4m+1$,

$k_1 = 4$, $b_1 = m\ (4m+1)$, $k_2 = 2$. Arrange the following m initial blocks into single block

$$\begin{bmatrix} x^i, x^{i+2m} \\ x^{i+m}, x^{i+3m} \end{bmatrix}, i = 0, 1, 2, \ldots m-1$$

By making pair of crosses in a single block and developing mod (p), we get double cross design with parameters $p = 4m+1$, $b = 4m+1$, $k = k_1/2$.

Note1: For construction of double cross design it is necessary that the block size of a single block must be an even number i.e. m must be a multiple of 2.

The procedure to obtain the above designs has been explained by the following illustrative example.

Example 1: If we let m = 2, we get the following two blocks.

$$\begin{bmatrix} (1,2) & (x,2x) \\ (2x+1, x+2) & (2x+2, x+1) \end{bmatrix}$$

We can now write both blocks in a single block as given below.

$$\begin{bmatrix} (1,2) \\ (2x+1, x+2) \\ (x,2x) \\ (2x+2, x+1) \end{bmatrix}$$

where x is a primitive element of GF(3²). Now cross the elements of the individual block and put these crosses in a single block. Adding successively the non-zero elements of GF (3²) to the contents of the single block, we obtain block and row- column design for partial double cross experiment with parameters p= 9, b= 9, k= 2. The design is exhibited below, where the lines have been relabelled 1-9, using the correspondence 0→1, 1→2, 2→3, x→4, x+1→5, x+2→6, 2x→7, 2x+1→8, 2x+2→9:

Partial Double Cross Row-Column Design

	B₁	B₂	B₃	B₄	B₅	B₆	B₇	B₈	B₉
R₁	(2×3)×(6×8)	(1×3)×(4×9)	(1×2)×(5×7)	(5×6)×(2×9)	(4×6)×(3×7)	(4×5)×(1×8)	(8×9)×(3×5)	(7×9)×(1×6)	(7×8)×(2×4)
R₂	(4×7)×(5×9)	(5×8)×(6×7)	(6×9)×(4×8)	(1×7)×(3×8)	(2×8)×(1×9)	(3×9)×(2×7)	(1×4)×(2×6)	(2×5)×(3×4)	(3×6)×(1×5)

Series 2: Let $p = 6m+1$, $m \geq 1$ be a prime or a prime power and x be a primitive element of the Galois field of order p. Consider the initial blocks

$$\left\{\left(x^i, x^{i+3m}\right), \left(x^{i+m}, x^{i+4m}\right), \left(x^{i+2m}, x^{i+5m}\right)\right\}, i = 0, 1, 2, \ldots m-1.$$

Dey et al. (1986)[4] showed when these initial blocks developed over mod (p), give a solution of a nested incomplete block design with parameters $v = p = 6m+1$, $b_1 = m(6m+1)$, $k_1 = 6$, $k_2 = 2$, $\lambda_2 = 1$.

We can then arrange the above initial blocks into a single block as given below

$$\begin{bmatrix} (x^i, x^{i+3m}) \\ (x^{i+m}, x^{i+4m}), i = 0, 1, \ldots, m-1 \\ (x^{i+2m}, x^{i+5m}) \end{bmatrix}$$

Example 2: Let m = 2. Then we get the following two initial blocks.

$$\begin{bmatrix} (1,12) & (2,11) \\ (4,9) & (8,5) \\ (3,10) & (6,7) \end{bmatrix}$$

We can arrange these two blocks in a single block as given below.

$$\begin{bmatrix} (1,12) \\ (2,11) \\ (4,9) \\ (8,5) \\ (3,10) \\ (6,7) \end{bmatrix}$$

Now performing crosses in pairs and developing these crosses over mod(p), we obtain block and row-column design with parameters p =13, b = 13 and k =3

$$\begin{bmatrix} (1,12) \\ (4,9) \\ (3,10) \\ (2,11) \\ (8,5) \\ (6,7) \end{bmatrix}$$

Partial Double Cross Row-Column Design

	B₁	B₂	B₃	B₄	B₅	B₆
R₁	(1×12)×(4×9)	(2×0)×(5×10)	(3×1)×(6×11)	(4×2)×(7×12)	(5×3)×(8×0)	(6×4)×(9×1)
R₂	(3×10)×(2×11)	(4×11)×(3×12)	(5×12)×(4×0)	(6×0)×(5×1)	(7×1)×(6×2)	(8×2)×(7×3)
R₃	(8×5)×(6×7)	(9×6)×(7×8)	(10×7)×(8×9)	(11×8)×(9×10)	(12×9)×(10×11)	(0×10)×(11×12)

	B₇	B₈	B₉	B₁₀	B₁₁	B₁₂	B₁₃
R₁	(7×5)×(10×2)	(8×6)×(11×3)	(9×7)×(12×4)	(10×8)×(0×5)	(11×9)×(1×6)	(0×10)×(2×7)	(1×11)×(3×8)
R₂	(9×3)×(8×4)	(10×4)×(9×5)	(11×5)×(10×6)	(12×6)×(11×7)	(0×7)×(12×8)	(1×8)×(0×9)	(2×9)×(1×10)
R₃	(1×11)×(12×0)	(2×12)×(0×1)	(3×0)×(1×2)	(4×1)×(2×3)	(5×2)×(3×4)	(6×3)×(4×5)	(7×4)×(5×6)

Series 3: Let $p = 2m+1$, $m \geq 2$ be a prime or a prime power and consider the following m blocks

(0, 2m), (1, 2m-1), (2, 2m-2), . . ., (m-1, m+1) mod (2m+1)

Example 3: If we let m= 4, then the single block will be as given below.

$$\begin{bmatrix} (0,8) \\ (1,7) \\ (2,6) \\ (3,5) \end{bmatrix}$$

Now applying the procedure given in example 1, we can obtain an optimal block design for double cross experiment with parameters p = 9, k= 2, b = 9.

Note 2: The m blocks given in series 3 form a nested balanced incomplete block design with parameters $v = p = 2m +1$, $b_1 = m$, $k_1 = m(2m+1)$, $k_2 = 2$, $\lambda_2 = 1$ given by Dey et al.[9].

Example: 4 Consider MRP 33 (2001) design with initial blocks given by

[{(∞ 0) (5 10)}{(1 2) (4 8)}{(6 9) (7 13)}{11 3) (12 14)}] mod 15.

Arranging these initial blocks in a single block and performing crosses between sub-blocks and developing mod(15), treatment ∞ is invariant under cyclic development of the initial blocks and we get optimal partial double cross block design with parameters p = 16, b =15, and k =4.

$$\begin{bmatrix} (\infty \times 0) \times (5 \times 10) \\ (1 \times 2) \times (4 \times 8) \\ (6 \times 9) \times (7 \times 13) \\ (11 \times 3) \times (12 \times 14) \end{bmatrix} mod \ (15)$$

Discussion

Universally optimal partial double cross design with p≤16, s ≤30 obtained by the above method from NBIB designs of Morgan, Preece& Rees [5], are listed in the following Table 1. These are the designs other than the designs catalogued by Das, Dey & Dean [12] and Parsad, et al.[4]. These are the new designs and successfully can be used in agricultural experimentation.

Table 1: Universally optimal block design for double cross with p ≤ 16, s ≤ 30 generated by using NBIB designs of Morgan et al. [5].

S.No	p	b	k	Source
1	16	15	4	MPR 33
2	12	33	5	MPR53
3	9	9	2	MPR 5w
4	9	9	2	MPR8
6	11	11	5	MPR49
7	13	13	3	MPR20w
8	13	13	3	MPR 21
9	13	13	3	MPR 23
10	14	14	7	MPR57
11	15	16	4	MPR33w
12	15	15	7	MPR59
13	13	39	2	MPR55
14	15	35	3	MPR62

Conclusion

We have presented a method of construction of three series of optimal block and row-column designs for partial double cross by using nested balanced incomplete block designs. Our proposed optimal block and row-column designs for partial double cross are new and not available in statistical literature.

Acknowledgement

None

Conflict of Interest

No conflict of interest.

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